

Homework 2 (due Monday Feb 15)

1. In this question we provide two websites. One on propositional calculus: <http://www.informatik.htw-dresden.de/~nestleri/logic/fundls.html> and one on circuits http://www.allaboutcircuits.com/vol_4/chpt_7/index.html

The first one goes through the material we have covered and it provides

Quizzes and self checking. You should spend a few hours on this.

The second one is about boolean algebra and circuits. One of the questions

In the midterm will be from this site.

2. This question is about knights and knaves. The knights always tell the truth, the knaves always lie. There are fascinating puzzles around this simple concept. Before you attempt any problem, read the material in:

<http://www.ux1.eiu.edu/~cfbxb/class/1900/formal/ch3/knkn/knight.htm>

http://en.wikipedia.org/wiki/Knights_and_Knaves

(a) *On an island, the populace is of two kinds: knights and knaves. Knights always tell the truth, knaves always lie. An islander - call him A - made a statement about himself and a friend, call him B: "Either I am a knave or B is a knight." What are A and B?*

(b) *As you approach the island, you spot three inhabitants on the shore. You call out to them, "Are you Knights or Knaves?" The first says something but you do not hear what he says, so you ask, "What did you say?" The second inhabitant says, "He says he is a Knight, he is and so am I." The third responds, "He is a Knave, but I am a Knight." What are the three inhabitants really?*

© In this website <http://www.hku.hk/cgi-bin/philodep/knight/puzzle> you will find 382 puzzles like the above generated by a computer. Try to solve at least five.

(d) **Challenge Problem:** Twenty-five knights and knaves sit around a round table. They all know each other well, but I don't know any of them. But I know that there's at least one knight at the table. I ask them all to point to the next knight on their left. Each one of them points to another person at the table. To my disappointment, even after carefully considering their answers, there is not a single person at the table who is certainly a knight, or certainly a knave. However, I now know how many knights are at the table in total!

3. Construct circuits corresponding to the following Boolean expressions

(a) $(p' + q)(p + (qr))$

(b) $((p + q)z)'$

© $(p'q + (q'r))' + ps'$

4. Use the rules of inference to show that the following arguments are valid:

(a) $\sim (s \wedge t), \sim w \rightarrow t$ implies $s \rightarrow w$

(b) $\sim (\sim p \vee q), \sim z \rightarrow \sim s, (p \wedge \sim q) \rightarrow s, \sim z \vee r$ implies r

5. The city council contains five members. Each member has a yes button and a no button for voting. An issue passes if the majority vote for it. Construct a circuit diagram which would determine whether or not an issue passes by having a light come on if the issue passes.

6. In the islands of Langerhans, a peculiar form of logic is employed. There are only two operators, a unary operator $\sim$ and a binary operator $*$, and one rule of inference, Reductio ad nauseum (or R for short):

R: Given $(\sim A)*B$ and B , we can conclude A .

There is also one axiom, Producia desde nil (or P).

P: $\sim(\sim A*B)*(B*C)$

Prove the following, using the axiom and the rules of inference:

- (a) $\sim D*(\sim(\sim A*B))$
- (b) $\sim E*\sim D$
- (c) E

7. **Challenge Problem:** You have been captured by cannibals. The head cannibal puts you in a room with two doors. He tells you: "One of these doors leads to death, the other to freedom. I leave here with you two of my men. One always lies and the other always says the truth. You are allowed to ask one of them one question. But your question to one of the men cannot involve the other." What is the question that will lead you to freedom?

8. Using the rules of inference, find conclusions for the following premises:

To wear a bow tie, it is necessary to attend a party. Those who do not live on the coast do not act silly. To look intelligent, one must wear a bow tie. No bassoon players chew gum. People who attend parties act silly. People who do not chew gum look intelligent.