Problem 1. Prove using mathematical induction, the sum of first $N$ positive even numbers is $N(N + 1)$.

Base case: When $N = 1$, the first even number is 2. $N(N + 1) = 1(1+1) = 2$. Verified.

Let us assume it is true for the first $k$ even numbers. That is, 
\[ \sum_{i=1}^{k} 2i = k(k + 1) \]

We want to prove that the sum of first $(k + 1)$ even numbers would then be given by $(k + 1)(k + 2)$.

The sum of first $(k + 1)$ even numbers is:

\[ \sum_{i=1}^{k+1} 2i = \sum_{i=1}^{k} 2i + 2(k + 1) \] (since the $(k + 1)^{th}$ even number is $2(k + 1)$.

Substituting, we have
\[ \sum_{i=1}^{k+1} 2i = k(k + 1) + 2(k + 1) = (k + 1)(k + 2), \] hence proved.

Therefore we have proved it by mathematical induction.
Problem 2.

def insertsort(a)
    a.length.times { |i|
        cur_elem = a[i]
        sorted_arr_len = i
        j = sorted_arr_len - 1
        while (j >= 0 and a[j] > cur_elem)
            a[j+1] = a[j]
            j = j - 1
        end
        a[j+1] = cur_elem
    }
    return a
end

arr = Array[3,0,-2,5,-3]
arr_sorted = insertsort(arr)

Given the above Ruby program, please trace the state of the array ‘a’ at the end of each iteration of the ‘times’ loop. In the above program there should be 5 iterations of the ‘times’ loop since the length of array ‘a’ is 5.

End of Iteration 1: Array a = [3,0,-2,5,-3]
End of Iteration 2: Array a = [0,3,-2,5,-3]
End of Iteration 3: Array a = [-2,0,3,5,-3]
End of Iteration 4: Array a = [-2,0,3,5,-3]
End of Iteration 5: Array a = [-3,-2,0,3,5]