Problem 1. Prove using induction that for all \( N \geq 1 \), the following holds:
\[
\sum_{i=1}^{N} i^2 = \frac{N(N+1)(2N+1)}{6}
\]

Problem 2. Give a 3 coloring of the following graph. Use letters ‘R’, ‘G’ and ‘B’ to denote the three colors and label the nodes with these letters.

Problem 3. A tree is a simple connected graph with no cycles. Specifically, in a tree of \( N \) nodes or vertices, there are \( N - 1 \) edges. A leaf node or vertex is a vertex on a tree which has degree 1.

(a) Prove (using induction or otherwise) that every tree has 2 or more leaf nodes.

(b) Using the above fact, prove using induction that every tree has a 2 coloring.

Problem 4. Prove using induction that the sum of first \( N \) (positive) even numbers is \( N(N + 1) \).