Homework 2

Due **in class** on Wed, Feb 29, 2012

This short homework assignment will help you review your understanding of the simply typed lambda calculus, extended with integers:

$$\begin{array}{rcl} e & ::= & v \mid x \mid e\ e \\ v & ::= & n \mid \lambda x.e \\ t & ::= & int \mid t \to t \\ A & ::= & \cdot \mid x : t, A \end{array}$$

1. Here are small-step, call-by-value operational semantics for lambda calculus:

$$\textsc{Beta}\ \frac{}{(\lambda x.e_1)v_2 \to e1[x \mapsto v_2]} \qquad \textsc{Left}\ \frac{e_1 \to e_1'}{e_1\ e_2 \to e_1'\ e_2} \qquad \textsc{Right}\ \frac{e_2 \to e_2'}{v\ e_2 \to v\ e_2'}$$

   (a) Draw derivations showing that the following reductions hold:
      i. $(\lambda x.42)\ 13 \to 42$
      ii. $((\lambda x.x)\ (\lambda y.y))\ (\lambda z.z) \to (\lambda y.y)\ (\lambda z.z)$
      iii. $(\lambda x.x)\ ((\lambda x.\lambda y.x)\ 42) \to (\lambda x.x)\ (\lambda y.42)$

   (b) Give a reduction that is possible in the fully non-deterministic operational semantics for lambda calculus, but which is not possible in the call-by-value operational semantics.

2. Here are the type rules for the simply typed lambda calculus:

$$\textsc{Int}\ \frac{}{A \vdash n : int} \qquad \textsc{Var}\ \frac{x \in dom(A)}{A \vdash x : A(x)} \qquad \textsc{Lam}\ \frac{x : t, A \vdash e : t'}{A \vdash \lambda x.e : t \to t'} \qquad \textsc{App}\ \frac{A \vdash e_1 : t \to t' \qquad A \vdash e_2 : t}{A \vdash e_1\ e_2 : t'}$$

   (a) Draw derivations showing that the following typing judgments hold:
      i. $\cdot \vdash 42 : int$
      ii. $y : int \vdash \lambda x.y : int \to int$
      iii. $\cdot \vdash \lambda x.\lambda y.x : int \to int \to int$
      iv. $+ : int \to int \to int \vdash (\lambda f.f\ 42)\ (\lambda x. +\ x\ 3) : int$

   (b) Write down an expression in the simply typed lambda calculus with integers that does not get "stuck" in under the operational semantics in part 1 (meaning, it will either run forever or reduce to a value), and yet is ill-typed. (Note that you do not have if...then...else available to you in this language as a built-in.)

3. By giving a counterexample, show that the following statement, which is essentially the reverse of Preservation, is false: If $A \vdash e' : t$ and $e \to e'$, then $A \vdash e : t$.