Spring 2012 **CMSC 451: Homework 5** Clyde Kruskal

Due Thursday, May 10, 2012.

Problem 1. Assume that you knew that 4-SAT was NP-complete, but did not know anything about 3-SAT. Show that

$$\text{4-SAT} \leq_{\text{P}} \text{Independent Set}$$

Problem 2. Consider the problem of *load balancing* on two processors: We have a set of n jobs, where job i takes time t_i. We would like to run them on two processors so that they finish as soon as possible.

(a) Define a decision version of this problem.

(b) Show the decision problem is in NP.

(c) Show the decision problem is complete for NP (that is, it is NP-hard). HINT: Reduction from subset sum.

(d) Show the decision problem is still complete for NP when there are three processors (that is, it is NP-hard).

Problem 3. Consider the problem DENSE SUBGRAPH: Given G, does it contain a subgraph H that has exactly K vertices and at least Y edges? Prove that this problem is NP-complete.

Problem 4. Consider the optimization problem of finding the largest number of clauses that can be satisfied in a 3-CNF formula, and the assignment to the variables that does so. Call the problem *max-3-SAT*.

(a) Define a decision version of the max-3-SAT.

(b) Show that the decision version is in NP.

(c) Show that the decision version is complete for NP (that is, it is NP-hard).

(d) Show that if you could solve the optimization version in polynomial time that you could also solve the decision version in polynomial time.

(e) Show that if you could solve the decision version in polynomial time that you could also solve the optimization version in polynomial time. HINT: First find the maximum number of clauses that can be satisfied.

(f) Show that you can solve the decision version for a constant number of clauses in polynomial time.