

CMSC 351: Practice Questions for Final Exam

These are practice problems for the upcoming final exam. You will be given a sheet of notes for the exam. Also, go over your homework assignments. **Warning:** This does not necessarily reflect the length, difficulty, or coverage of the actual exam.

Problem 1.

- (a) Is $2^{n+1} = O(2^n)$?
- (a) Is $2^{2n} = O(2^n)$?

Problem 2. Assume you have a list of n elements where every number is within k positions of its correct location, for some constant k .

- (a) Give an algorithm that sorts this list with as few comparisons as possible (as a function of n and k). Just get the high order term right. How many comparisons does your algorithm use?
- (b) Show that your algorithm is optimal using a decision tree argument.

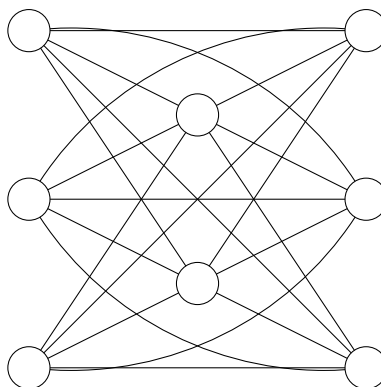
Problem 3. Let $A[1, \dots, n]$ be an array of n numbers (some positive and some negative).

- (a) Give an algorithm to find which *three* numbers have sum closest to zero. Make your algorithm as efficient as possible. Write it in pseudo-code.
- (b) Analyze its running time.

Problem 4. Assume that you developed an algorithm to find the (index of the) $n/3$ smallest element of a list of n elements in $2n$ comparisons.

- (a) Using the algorithm (as a black box), give an algorithm, efficient in the worst case, to find the k th smallest element of a list.
- (b) Write down a recurrence for (a bound on) the number of comparisons it executes in the worst case.
- (c) Solve the recurrence (using constructive induction). Find the high order term exactly (but you do not need any low order terms).
- (d) Using the (black box) algorithm for finding the $n/3$ smallest element and using the ideas and results of Parts (a), (b), and (c), give an efficient algorithm to find (the index of) two elements, the k_1 th smallest and the k_2 smallest (for inputs k_1 and k_2). The algorithm description can be very high level and brief.
- (e) How many comparisons does it use? Find the high order term exactly (but you do not need any low order terms). Give a brief justification.

Problem 5. A graph is tripartite if the vertices can be partitioned into three sets so that there are no edges internal to any set. The *complete* tripartite graph, $K(a, b, c)$, has three sets of vertices with sizes a , b , and c and all possible edges between each pair of sets of vertices. $K(3, 2, 3)$ is pictured below. A *Hamiltonian* cycle in a graph is a cycle that traverses every vertex exactly once.



- (a) For which values of n does $K(1, 1, n)$ have a Hamiltonian cycle. Justify your answer.
- (b) For which values of n does $K(1, n, n)$ have a Hamiltonian cycle. Justify your answer.
- (c) For which values of n does $K(n, n, n)$ have a Hamiltonian cycle. Justify your answer.

Problem 6. Let $G = (V, E)$ be an undirected graph. A *triangle* is a set of three vertices such that each pair has an edge.

- (a) Give an efficient algorithm to find all of the triangles in a graph.
- (b) How fast is your algorithm?

Problem 7. This problem is more open-ended than you would see on an exam: If you do not know how to play Sudoku, look it up. Normally, Sudoku is played on a 9×9 grid.

- (a) Generalize Sudoku to larger grids.
- (b) State the (generalized) Sudoku game as a decision problem.
- (c) Show that the decision version of (generalized) Sudoku is in NP.
- (d) Show that if you can solve the decision version of (generalized) Sudoku in polynomial time, you can solve a (generalized) Sudoku puzzle in polynomial time.