

Due Thursday, March 7. Write your NAME and SECTION on the top of your solution.

Problem 1. Evaluate $\prod_{k=1}^n (4 \cdot 3^k)$. Show your work.

Problem 2. Use a non-integral method to get upper and lower bounds for $\sum_{j=0}^n \sqrt{j}$ that are within a constant factor of each other. Show your work.

Problem 3. Use the integral method to get upper and lower bounds for $\sum_{j=0}^n \sqrt{j}$. The two values should have the same high level term. Show your work.

Problem 4. Assume that you derive that the number of comparisons in Merge Sort satisfies $M(n) = 2M(n/2) + n - 1$ and $M(1) = 0$ for n a power of 2. Assume you guess that $M(n) = an \lg n + bn + c$ for constants a, b, c . Use constructive induction to prove this and derive values for a, b, c . Show your work.

Problem 5. We showed in class (and in the book) that $\sum_{i=1}^n \frac{1}{i} \leq \lg n + O(1)$. Using the same method, show how to split each interval in half to get a better bound. Show your work.