Due Thursday, May 9, 2013.

**Problem 1.** The *Crossing Hamiltonian Cycle* problem (CHP) is: Given an undirected graph $G = (V, E)$, does there exist a cycle that passes through every vertex of $G$ and crosses itself exactly once? (That is, it starts at a vertex, visits some vertices, comes back to the original vertex, visits the rest of the vertices, and finally ends at the original vertex. Think figure eight.) Show that the Crossing Hamiltonian Cycle problem is in NP.

**Problem 2.** Assume that you have a polynomial time algorithm that determines if a Boolean formula is satisfiable. Show that, given a Boolean formula that is satisfiable, you could use the algorithm to find a satisfying assignment in polynomial time.

**Problem 3.** A *Safe Set of Vertices* is a set of vertices in an undirected graph $G = (V, E)$ such that no two vertices in the set are within distance 2. (That is the shortest path between any pair of vertices in the safe set uses at least three edges.) Assume that the vertices of a graph have weights. The *Weighted Safe Set of Vertices Problem* is to find a safe set of vertices whose sum of weights is as large as possible.

(a) Define a decision version of the weighted safe set of vertices problem.

(b) Show that the decision version of the weighted safe set of vertices problem is in NP.

(c) Show that if you could solve the optimization version in polynomial time that you could also solve the decision version in polynomial time.

(d) **Challenge Problem.** Show that if you could solve the decision version in polynomial time that you could also solve the optimization version in polynomial time. Note: There are two steps here. First find the value of largest weighted safe set, and then find the vertices themselves.