

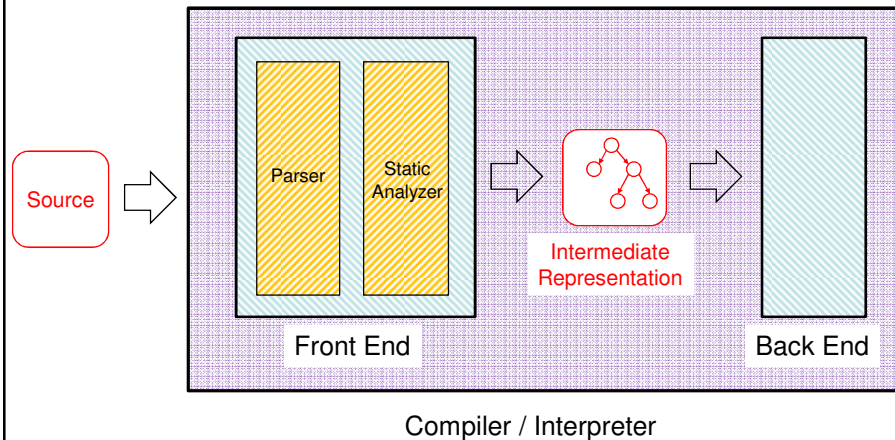
CMSC 330: Organization of Programming Languages

Context Free Grammars

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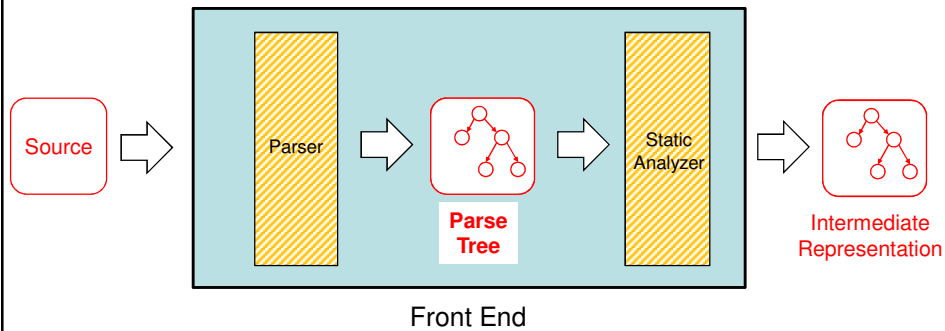
Architecture of Compilers, Interpreters



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Front End

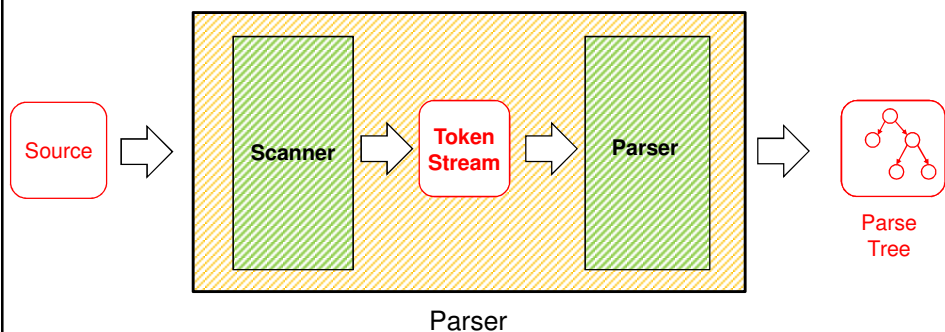


- ▶ Responsible for turning source code (sequence of symbols) into representation of program structure
- ▶ Parser generates **parse trees**, which static analyzer processes

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Parser Architecture



- ▶ **Scanner / lexer** converts sequences of symbols into **tokens** (keywords, variable names, operators, numbers, etc.)
- ▶ **Parser** (yes, a parser contains a parser!) converts sequences of tokens into parse tree; implements a **context free grammar**
 - We can't do everything as a regular expression – not powerful enough

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Context Free Grammar (CFG)

- ▶ A way of describing **sets of strings** (= languages)
 - The notation $L(S)$ denotes the language of strings defined by S
- ▶ Example grammar: $S \rightarrow 0S \mid 1S \mid \epsilon$
String $s' \in L(S)$ iff
 - $s' = \epsilon$, or $\exists s \in L(S)$ such that $s' = 0s$, or $s' = 1s$
- ▶ Grammar is same as regular expression $(0|1)^*$
 - Generates / accepts the same set of strings

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Context-Free Grammars (CFGs)

- ▶ But CFGs can do what regexps cannot
 - $S \rightarrow (S) \mid \epsilon$ // represents balanced pairs of $()$'s
- ▶ In fact, CFGs subsume REs, DFAs, NFAs
 - There is a CFG that generates any regular language
 - But REs are a better notation for regular languages
- ▶ CFGs can specify programming language syntax
 - CFGs (mostly) describe the parsing process

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Formal Definition: Context-Free Grammar

- ▶ A CFG G is a 4-tuple (Σ, N, P, S)
 - Σ – alphabet (finite set of symbols, or terminals)
 - Often written in lowercase
 - N – a finite, nonempty set of nonterminal symbols
 - Often written in uppercase
 - It must be that $N \cap \Sigma = \emptyset$
 - P – a set of productions of the form $N \rightarrow (\Sigma|N)^*$
 - Informally: the nonterminal can be replaced by the string of zero or more terminals / nonterminals to the right of the $\rightarrow$
 - Can think of productions as rewriting rules (more later)
 - $S \in N$ – the start symbol

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Backus-Naur Form

- ▶ Context-free grammar production rules are also called Backus-Naur Form or BNF
 - A production like $A \rightarrow B c D$ is written in BNF as $\langle A \rangle ::= \langle B \rangle c \langle D \rangle$ (Non-terminals written with angle brackets and $::=$ instead of $\rightarrow$)
 - Often used to describe language syntax
- ▶ BNF was designed by
 - John Backus
 - Chair of the Algol committee in the early 1960s
 - Peter Naur
 - Secretary of the committee, who used this notation to describe Algol in 1962

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Generating Strings

- ▶ We can think of a grammar as **generating** strings by rewriting
- ▶ Example grammar: $S \rightarrow 0S \mid 1S \mid \epsilon$
 - $S \Rightarrow 0S$ // using $S \rightarrow 0S$
 - $\Rightarrow 01S$ // using $S \rightarrow 1S$
 - $\Rightarrow 011S$ // using $S \rightarrow 1S$
 - $\Rightarrow 011$ // using $S \rightarrow \epsilon$

Accepting Strings (Informally)

- ▶ Determining if $s \in L(S)$ is called **acceptance**: goal is to find a rewriting from S that yields s
 - $011 \in L(S)$ according to the previous rewriting
 - A rewriting is some sequence of productions (**rewrites**) applied starting at the start symbol
- ▶ Terminology
 - Such a sequence of rewrites is a **derivation** or **parse**
 - Discovering the derivation is called **parsing**

Derivations

► Notation

- $\Rightarrow$ indicates a derivation of one step
- $\Rightarrow^+$ indicates a derivation of one or more steps
- $\Rightarrow^*$ indicates a derivation of zero or more steps

► Example

- $S \rightarrow 0S \mid 1S \mid \epsilon$

► For the string 010

- $S \Rightarrow 0S \Rightarrow 01S \Rightarrow 010S \Rightarrow 010$
- $S \Rightarrow^+ 010$
- $S \Rightarrow^* S$

Language Generated by Grammar

► $L(G)$ the language defined by G is

$$L(G) = \{ \omega \in \Sigma^* \mid S \Rightarrow^+ \omega \}$$

- S is the start symbol of the grammar
- Σ is the alphabet for that grammar

► In other words

- All strings over Σ that can be derived from the start symbol via one or more productions

Practice

- ▶ Try to make a grammar which accepts
 - 0^*1^* – 0^n1^n where $n \geq 0$ – 0^n1^m where $m \leq n$
- $S \rightarrow A \mid B$
 $A \rightarrow 0A \mid \epsilon$ $S \rightarrow 0S1 \mid \epsilon$ $S \rightarrow 0S1 \mid 0S \mid \epsilon$
 $B \rightarrow 1B \mid \epsilon$
- ▶ Give some example strings from this language
 - $S \rightarrow 0 \mid 1S$
 - 0, 10, 110, 1110, 11110, ...
 - What language is it?
 - 1^*0

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Example: Arithmetic Expressions

- ▶ $E \rightarrow a \mid b \mid c \mid E+E \mid E-E \mid E^*E \mid (E)$
 - An expression E is either a letter a , b , or c
 - Or an E followed by $+$ followed by an E
 - etc...
- ▶ This **describes** (or **generates**) a set of strings
 - $\{a, b, c, a+b, a+a, a^*c, a-(b^*a), c^*(b+a), \dots\}$
- ▶ Example strings not in the language
 - $d, c(a), a+, b^{**}c$, etc.

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Formal Description of Example

- Formally, the grammar we just showed is
 - $\Sigma = \{ +, -, *, (,), a, b, c \}$ // terminals
 - $N = \{ E \}$ // nonterminals
 - $P = \{ \begin{array}{l} E \rightarrow a, E \rightarrow b, E \rightarrow c, \\ E \rightarrow E-E, E \rightarrow E+E, \\ E \rightarrow E*E, \\ E \rightarrow (E) \end{array} \}$ // productions
 - $S = E$ // start symbol

(Non-)Uniqueness of Grammars

- Different grammars generate the same set of strings (language)
- The following grammar generates the same set of strings as the previous grammar

$E \rightarrow E+T \mid E-T \mid T$

$T \rightarrow T*P \mid P$

$P \rightarrow (E) \mid a \mid b \mid c$

Notational Shortcuts

- ▶ A production is of the form
 - left-hand side (LHS) $\rightarrow$ right hand side (RHS)
- ▶ If not specified
 - Assume LHS of first listed production is the start symbol
- ▶ Productions with the same LHS
 - Are usually combined with |
- ▶ If a production has an empty RHS
 - It means the RHS is ϵ

$S \rightarrow ABC$	// S is start symbol
$A \rightarrow aA$	
b	// A $\rightarrow$ b
	// A $\rightarrow \epsilon$

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Practice

- ▶ Given the grammar
$$\begin{aligned} S &\rightarrow aS \mid T \\ T &\rightarrow bT \mid U \\ U &\rightarrow cU \mid \epsilon \end{aligned}$$
- Provide derivations for the following strings
 - ▶ b $S \Rightarrow T \Rightarrow bT \Rightarrow bU \Rightarrow b$
 - ▶ ac $S \Rightarrow aS \Rightarrow aT \Rightarrow aU \Rightarrow acU \Rightarrow ac$
 - ▶ bbc $S \Rightarrow T \Rightarrow bT \Rightarrow bbT \Rightarrow bbU \Rightarrow bbcU \Rightarrow bbc$
- Does the grammar generate the following?
 - ▶ $S \Rightarrow^+ ccc$ Yes $S \Rightarrow^+ bS$ No
 - ▶ $S \Rightarrow^+ bab$ No $S \Rightarrow^+ Ta$ No

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Practice

- ▶ Given the grammar

$S \rightarrow aS \mid T$

$T \rightarrow bT \mid U$

$U \rightarrow cU \mid \epsilon$

- Name language accepted by grammar

➤ $a^*b^*c^*$

- Give a different grammar accepting language

$S \rightarrow ABC$

$A \rightarrow aA \mid \epsilon$ // a^*

$B \rightarrow bB \mid \epsilon$ // b^*

$C \rightarrow cC \mid \epsilon$ // c^*

Parse Trees

- ▶ Parse tree shows how a string is produced by a grammar

- Root node is the start symbol
- Every internal node is a nonterminal
- Children of an internal node
 - Are symbols on RHS of production applied to nonterminal
- Every leaf node is a terminal or ϵ

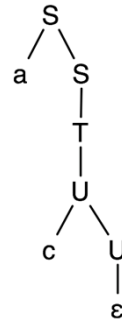
- ▶ Reading the leaves left to right

- Shows the string corresponding to the tree

Parse Tree Example

$S \Rightarrow aS \Rightarrow aT \Rightarrow aU \Rightarrow acU \Rightarrow ac$

$S \rightarrow aS \mid T$
 $T \rightarrow bT \mid U$
 $U \rightarrow cU \mid \epsilon$



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Parse Trees for Expressions

- A **parse tree** shows the structure of an expression as it corresponds to a grammar

$E \rightarrow a \mid b \mid c \mid d \mid E+E \mid E-E \mid E^*E \mid (E)$

a

```

graph TD
    E1[E] --- a[a]
  
```

a^*c

```

graph TD
    E1[E] --- E2[E]
    E1 --- star[*]
    E1 --- E3[E]
    E2 --- a[a]
    E3 --- c[c]
  
```

$c^*(b+d)$

```

graph TD
    E1[E] --- E2[E]
    E1 --- star1[*]
    E1 --- E3[E]
    E2 --- c[c]
    E3 --- LP["("]
    E3 --- E4[E]
    E3 --- plus1["+"]
    E3 --- RP[")"]
    E4 --- E5[E]
    E4 --- plus2["+"]
    E4 --- E6[E]
    E5 --- b[b]
    E6 --- d[d]
  
```

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Practice

$E \rightarrow a \mid b \mid c \mid d \mid E+E \mid E-E \mid E^*E \mid (E)$

Make a parse tree for...

- a^*b
- $a+(b-c)$
- $d^*(d+b)-a$
- $(a+b)^*(c-d)$
- $a+(b-c)^*d$

Leftmost and Rightmost Derivation

- ▶ Leftmost derivation
 - Leftmost nonterminal is replaced in each step
- ▶ Rightmost derivation
 - Rightmost nonterminal is replaced in each step
- ▶ Example
 - Grammar
 - $S \rightarrow AB, A \rightarrow a, B \rightarrow b$
 - Leftmost derivation for "ab"
 - $S \Rightarrow AB \Rightarrow aB \Rightarrow ab$
 - Rightmost derivation for "ab"
 - $S \Rightarrow AB \Rightarrow Ab \Rightarrow ab$

Parse Tree For Derivations

- Parse tree may be same for both leftmost & rightmost derivations

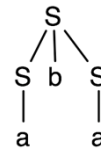
- Example Grammar: $S \rightarrow a \mid SbS$ String: aba

Leftmost Derivation

$S \Rightarrow SbS \Rightarrow abS \Rightarrow aba$

Rightmost Derivation

$S \Rightarrow SbS \Rightarrow Sba \Rightarrow aba$



- Parse trees don't show order productions are applied
- Every parse tree has a unique leftmost and a unique rightmost derivation

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Parse Tree For Derivations (cont.)

- Not every string has a unique parse tree

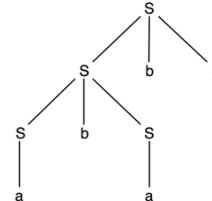
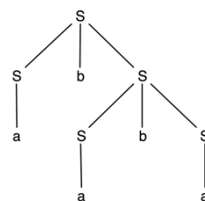
- Example Grammar: $S \rightarrow a \mid SbS$ String: $ababa$

Leftmost Derivation

$S \Rightarrow SbS \Rightarrow abS \Rightarrow abSbS \Rightarrow ababS \Rightarrow ababa$

Another Leftmost Derivation

$S \Rightarrow SbS \Rightarrow SbSbS \Rightarrow abSbS \Rightarrow ababS \Rightarrow ababa$



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Ambiguity

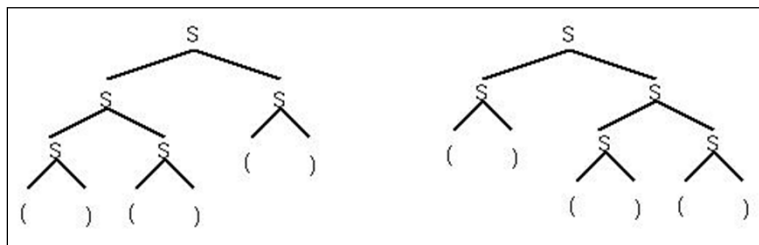
- ▶ A grammar is **ambiguous** if a string may have multiple **leftmost** derivations
 - Equivalent to multiple parse trees
 - Can be hard to determine
- | | |
|--|------------|
| 1. $S \rightarrow aS \mid T$ | |
| $T \rightarrow bT \mid U$ | No |
| $U \rightarrow cU \mid \epsilon$ | |
| 2. $S \rightarrow T \mid T$ | Yes |
| $T \rightarrow Tx \mid Tx \mid x \mid x$ | |
| 3. $S \rightarrow SS \mid () \mid (S)$ | ? |

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Ambiguity (cont.)

- ▶ Example
 - Grammar: $S \rightarrow SS \mid () \mid (S)$ String: $()()$
 - 2 distinct (leftmost) derivations (and parse trees)
 - $S \Rightarrow \underline{S}S \Rightarrow \underline{SS}S \Rightarrow ()\underline{SS} \Rightarrow ()()\underline{S} \Rightarrow ()()$
 - $S \Rightarrow \underline{S}S \Rightarrow ()\underline{S} \Rightarrow ()\underline{SS} \Rightarrow ()()\underline{S} \Rightarrow ()()$



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CFGs for Programming Languages

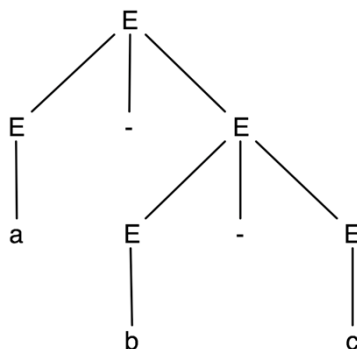
- ▶ Recall that our goal is to describe programming languages with CFGs
- ▶ We had the following example which describes limited arithmetic expressions
$$E \rightarrow a \mid b \mid c \mid E+E \mid E-E \mid E^*E \mid (E)$$
- ▶ What's wrong with using this grammar?
 - It's ambiguous!

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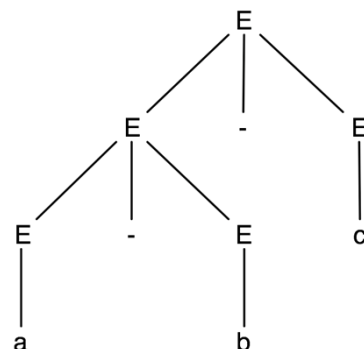
Example: a-b-c

$E \Rightarrow E-E \Rightarrow a-E \Rightarrow a-E-E \Rightarrow$
 $a-b-E \Rightarrow a-b-c$



Corresponds to $a-(b-c)$

$E \Rightarrow E-E \Rightarrow E-E-E \Rightarrow$
 $a-E-E \Rightarrow a-b-E \Rightarrow a-b-c$



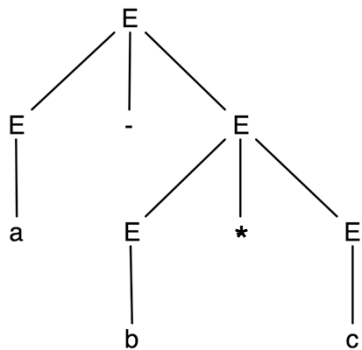
Corresponds to $(a-b)-c$

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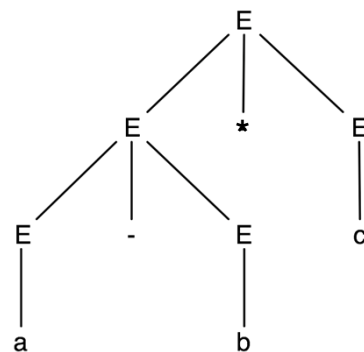
Example: $a-b^*c$

$E \Rightarrow E-E \Rightarrow a-E \Rightarrow a-E^*E \Rightarrow$
 $a-b^*E \Rightarrow a-b^*c$



Corresponds to $a-(b^*c)$

$E \Rightarrow E-E \Rightarrow E-E^*E \Rightarrow$
 $a-E^*E \Rightarrow a-b^*E \Rightarrow a-b^*c$



Corresponds to $(a-b)^*c$

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Another Example: If-Then-Else

$\langle \text{stmt} \rangle \rightarrow \langle \text{assignment} \rangle \mid \langle \text{if-stmt} \rangle \mid \dots$

$\langle \text{if-stmt} \rangle \rightarrow \text{if } (\langle \text{expr} \rangle) \langle \text{stmt} \rangle \mid$

$\text{if } (\langle \text{expr} \rangle) \langle \text{stmt} \rangle \text{ else } \langle \text{stmt} \rangle$

(Note $\langle \rangle$'s are used to denote nonterminals)

- Consider the following program fragment

if ($x > y$)

if ($x < z$)

a = 1;

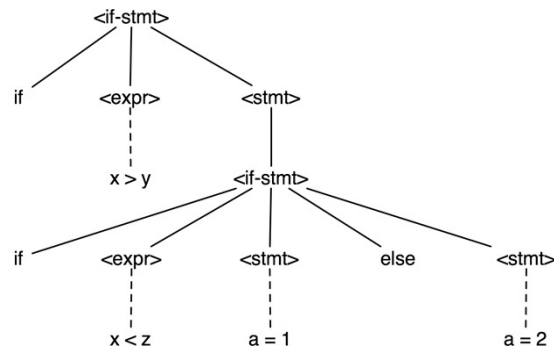
else a = 2;

(Note: Ignore newlines)

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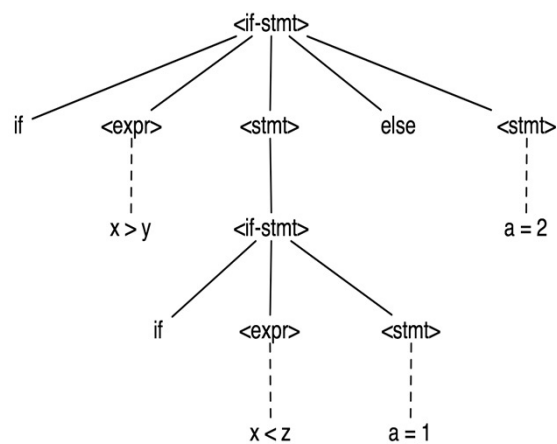
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Parse Tree #1



- Else belongs to inner if

Parse Tree #2



- Else belongs to outer if

Dealing With Ambiguous Grammars

- ▶ Ambiguity is bad
 - Syntax is correct
 - But semantics differ depending on choice
 - Different associativity (a-b)-c vs. a-(b-c)
 - Different precedence (a-b)*c vs. a-(b*c)
 - Different control flow if (if else) vs. if (if) else
- ▶ Two approaches
 - Rewrite grammar
 - Use special parsing rules
 - Depending on parsing method (learn in CMSC 430)

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Fixing the Expression Grammar

- ▶ Require right operand to not be bare expression

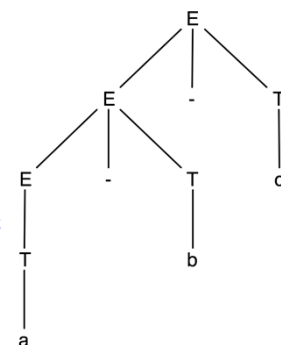
$E \rightarrow E+T \mid E-T \mid E*T \mid T$

$T \rightarrow a \mid b \mid c \mid (E)$

- ▶ Corresponds to **left associativity**

- ▶ Now only one parse tree for **a-b-c**

- Find derivation



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What If We Want Right Associativity?

- ▶ Left-recursive productions
 - Used for left-associative operators
 - Example
$$E \rightarrow E+T \mid E-T \mid E*T \mid T$$
$$T \rightarrow a \mid b \mid c \mid (E)$$
- ▶ Right-recursive productions
 - Used for right-associative operators
 - Example
$$E \rightarrow T+E \mid T-E \mid T^*E \mid T$$
$$T \rightarrow a \mid b \mid c \mid (E)$$

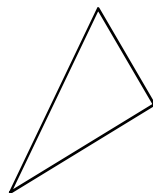
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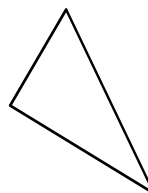
Parse Tree Shape

- ▶ The kind of recursion determines the shape of the parse tree

left recursion



right recursion



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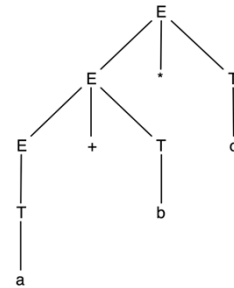
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A Different Problem

- ▶ How about the string $a+b*c$?

$E \rightarrow E+T \mid E-T \mid E*T \mid T$

$T \rightarrow a \mid b \mid c \mid (E)$



- ▶ Doesn't have correct precedence for $*$
 - When a nonterminal has productions for several operators, they effectively have the same precedence
- ▶ Solution – Introduce **new** nonterminals

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Final Expression Grammar

$E \rightarrow E+T \mid E-T \mid T$

lowest precedence operators

$T \rightarrow T*P \mid P$

higher precedence

$P \rightarrow a \mid b \mid c \mid (E)$

highest precedence (parentheses)

- ▶ Controlling precedence of operators
 - Introduce new nonterminals
 - Precedence increases closer to operands
- ▶ Controlling associativity of operators
 - Introduce new nonterminals
 - Assign associativity based on production form
 - ▶ $E \rightarrow E+T$ (left associative) vs. $E \rightarrow T+E$ (right associative)

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Tips For Designing Grammars

1. Use recursive productions to generate an arbitrary number of symbols
 $A \rightarrow xA \mid \epsilon$ // Zero or more x 's
 $A \rightarrow yA \mid y$ // One or more y 's
2. Use separate non-terminals to generate disjoint parts of a language, and then combine in a production

$\{a^*b^*\}$ // a 's followed by b 's
 $S \rightarrow AB$
 $A \rightarrow aA \mid \epsilon$ // Zero or more a 's
 $B \rightarrow bB \mid \epsilon$ // Zero or more b 's

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Tips For Designing Grammars (cont.)

3. To generate languages with matching, balanced, or related numbers of symbols, write productions which generate strings from the middle

$\{a^n b^n \mid n \geq 0\}$ // N a 's followed by N b 's
 $S \rightarrow aSb \mid \epsilon$

Example derivation: $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$

$\{a^n b^{2n} \mid n \geq 0\}$ // N a 's followed by $2N$ b 's
 $S \rightarrow aSbb \mid \epsilon$

Example derivation: $S \Rightarrow aSbb \Rightarrow aaSbbbb \Rightarrow aabbbb$

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Tips For Designing Grammars (cont.)

4. For a language that is the union of other languages, use separate nonterminals for each part of the union and then combine

$\{ a^n(b^m|c^m) \mid m > n \geq 0 \}$

Can be rewritten as

$\{ a^n b^m \mid m > n \geq 0 \} \cup \{ a^n c^m \mid m > n \geq 0 \}$

$S \rightarrow T \mid V$

$T \rightarrow aTb \mid U$

$U \rightarrow Ub \mid b$

$V \rightarrow aVc \mid W$

$W \rightarrow Wc \mid c$