CMSC 330: Organization of
Programming Languages

Examples of REs & Finite Automata

Describing Regular Expressions

a) $0(0|1)^*0$
   - All strings beginning and ending in 0
b) $((\epsilon|0)1^*)^*$
   - All strings
c) $(0|1)^*0(0|1)(0|1)$
   - All strings with 0 as third digit from right
Creating Regular Expressions

For all strings of 0’s and 1’s that...

a) Begin in 1
   - 1(0|1)*

b) End in 1
   - (0|1)*1

c) Contains 00
   - (0|1)*00(0|1)*

d) Do not contain 00
   - (01|1)*(ε|0)

Creating NFA

For all strings of 0’s and 1’s that...

a) Begin in 1

b) End in 1

c) Contains 00

d) Do not contain 00

Based on regular expression
Creating DFA

For all strings of 0’s and 1’s that…

a) Begin in 1

b) End in 1

c) Contains 00

d) Do not contain 00

Swap final / non-final states!

For RE (a | b )*

a) Construct NFA

b) Accept ababbab

7,5,1,2,6,8,7,5,3,4,6,8,7,5,1,2,6,8,7,5,3,4,6,8,7,5,3,4,6,
For RE (a | b )*

c) Reduce NFA to DFA
- Start = $\epsilon$-closure(7) = {7,5,1,3,8}
- $R = \{7,5,1,3,8\}$
- $r \in R = \{7,5,1,3,8\}$ // mark DFA state
- move (\{7,5,1,3,8\}, a) = \{2\}
  - $e = \epsilon$-closure(\{2\}) = \{2,6,8,7,5,1,3\} // new DFA state
  - $R = R \cup \{2,6,8,7,5,1,3\}$ // add to $R$
  - $\delta = \delta \cup (\{7,5,1,3,8\}, a, \{2,6,8,7,5,1,3\})$
- move (\{7,5,1,3,8\}, b) = \{4\}
  - $e = \epsilon$-closure(\{4\}) = \{4,6,8,7,5,1,3\} // new DFA state
  - $R = R \cup \{4,6,8,7,5,1,3\}$ // add to $R$
  - $\delta = \delta \cup (\{7,5,1,3,8\}, b, \{4,6,8,7,5,1,3\})$

For RE (a | b )*

- $R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}$
- $r \in R = \{2,6,8,7,5,1,3\}$ // mark DFA state
- move (\{2,6,8,7,5,1,3\}, a) = \{2\}
  - $e = \epsilon$-closure(\{2\}) = \{2,6,8,7,5,1,3\} // existing DFA state
  - $\delta = \delta \cup (\{2,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\})$
- move (\{2,6,8,7,5,1,3\}, b) = \{4\}
  - $e = \epsilon$-closure(\{4\}) = \{4,6,8,7,5,1,3\} // existing DFA state
  - $\delta = \delta \cup (\{2,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\})$
For RE \((a | b)^*\)

- \(R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}\)
- \(r \in R = \{4,6,8,7,5,1,3\}\)  // mark DFA state
- move \((\{4,6,8,7,5,1,3\}, a) = \{2\}\)
  - \(e = \varepsilon\)-closure\((\{2\}) = \{2,6,8,7,5,1,3\}\)  // existing DFA state
  - \(\delta = \delta \cup (\{4,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\})\)
- move \((\{4,6,8,7,5,1,3\}, b) = \{4\}\)
  - \(e = \varepsilon\)-closure\((\{4\}) = \{4,6,8,7,5,1,3\}\)  // existing DFA state
  - \(\delta = \delta \cup (\{4,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\})\)
- \(R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}\)
  - No more unmarked states to process
- \(F_d = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}\)
  - Since \(8 \in F_n\)

For RE \((a | b)^*\)

- Resulting DFA
  \[\Sigma = \{a, b\}\]
  \[R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}\]
  \[r_0 = \{7,5,1,3,8\}\]
  \[F_d = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}\]
  \[\delta = \{\]
  \((\{7,5,1,3,8\}, a, \{2,6,8,7,5,1,3\}), \]
  \((\{7,5,1,3,8\}, b, \{4,6,8,7,5,1,3\}), \]
  \((\{2,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\}), \]
  \((\{2,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\}), \]
  \((\{4,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\}), \]
  \((\{4,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\})\}\}
For RE \((a | b)^*\)

- NFA to DFA reduction pictorial

```
    7, 5, 1, 3, 8
     |  a
    2, 6, 8, 7, 5, 1, 3
     |  a  b
     v  a  b
4, 6, 8, 7, 5, 1, 3
```

**For RE \((a | b)^*\)**

d) Minimize DFA

- Initial partitions
  - Accept → \{1,2,3\} → P1
  - Reject → ∅

- Split partition?
  - move(1,a) → P1
  - move(2,a) → P1
  - move(3,a) → P1
  - move(1,b) → P1
  - move(2,b) → P1
  - move(3,b) → P1

Not required, minimization done
For RE \((a^* | b^*)^*\)

a) Construct NFA

b) Accept \(ababbab\)

\[11,9,3,1,2,4,10,12,11,9,7,5,6,8,10,12,11,9,3,1,2,4,10,12,…\]

For RE \((a^* | b^*)^*\)

c) Reduce NFA to DFA

- Start = \(\epsilon\)-closure(1) 
  \[= \{11,9,3,1,4,7,5,8,10,12\}\]
- \(R = \{\{11,9…12\}\}\)
- \(r \in R = \{11,9…12\}\)}
- move \((\{11,9…12\}, a) = 2\)
  \[\cdot \ \epsilon\text{-closure}(2) = \{2,11,9…12\}\]
- move \((\{11,9,…,12\}, b) = 6\)
  \[\cdot \ \epsilon\text{-closure}(6) = \{6,11,9…12\}\]
- ... 
- NFA to DFA reduction pictorial
For RE \((a^* \mid b^*)^*\)

d) Minimize DFA
- Initial partitions
  - Accept \(\rightarrow \{1,2,3\} \rightarrow P1\)
  - Reject \(\rightarrow \emptyset\)
- Split partition?
  - move(1,a) \(\rightarrow P1\)
  - ... Not required, minimization done

e) Compare 2 minimized DFAs
- Identical up to state names!