Reminders

- Where is the class webpage?
  - Announcements
  - Syllabus
  - Lecture Slides
  - TA Office hours schedule

- All students must attend lecture and discussion for which they are officially registered
- You are expected to attend every class session
- No electronic devices during class

Logical Equivalence

Recall: Two statements are **logically equivalent** if they have the same truth values for every possible interpretation.

Notation:

$p \equiv {\sim}{\sim}p$

How can we check whether or not two statements are logically equivalent?

Examples:

${\sim}(p \vee {\sim}q) \vee ({\sim}q \wedge {\sim}p) \overset{?}{\equiv} {\sim}p$

${\sim}p \vee {\sim}q \overset{?}{\equiv} {\sim}(p \vee q)$

Logical Equivalence

Show that $(p \wedge q \wedge r \wedge s) \vee t$ is **not** logically equivalent to $(p \wedge q \wedge r) \wedge (s \vee t)$

Must we draw a complete truth table with 32 rows?

Tautology and Contradiction

- A statement is a **tautology** if it is *true* under every possible interpretation.
- A statement is a **contradiction** if it is *false* under every possible interpretation.
- Examples

Equivalencies in Propositional Logic

Given any statement variables p, q, and r, a tautology t and a contradiction c, the following logical equivalences hold:		
1. Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws:	$p \wedge t \equiv p$	$p \vee c \equiv p$
5. Negation laws:	$p \vee \sim p \equiv t$	$p \wedge \sim p \equiv c$
6. Double negative law:	$\sim(\sim p) \equiv p$	
7. Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. DeMorgan's laws:	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
9. Universal bounds laws:	$p \vee t \equiv t$	$p \wedge c \equiv c$
10. Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negations of t and c:	$\sim t \equiv c$	$\sim c \equiv t$

- You don't need to memorize this
- Posted on class webpage (under "resources")
- We can substitute long expressions for the variables above
- Let's derive a few of these with truth tables

Simplifying Using the "Laws"

Let's use the "Laws of Equivalence" to simplify this sentence:

$\sim(\sim p \wedge q) \wedge (p \vee \sim q)$

Deriving Equivalencies Using "Laws"

Previously, we showed (using a truth table):

$$\sim(p \vee \sim q) \vee (\sim q \wedge \sim p) \equiv \sim p$$

Let's now demonstrate this equivalency a different way by using the established "laws" of equivalence.

Conditional Connective

$p \rightarrow q$ represents "If p then q" or "p implies q"

- If p is true then q must also be true
- If p is false then q could be either true/false

p	**q**	**$p \rightarrow q$**
1	1	1
1	0	0
0	1	1
0	0	1

- Note: precedence is **lower** than conjunction/disjunction
- Examples translating from English
- Differences between logical connective and everyday English

More Equivalencies

$p \rightarrow q \equiv {\sim}p \vee q$

${\sim}(p \rightarrow q) \equiv p \wedge {\sim}q$

Let's show one with truth tables, one with the equivalency "laws"

Definitions for Conditional Statements

The **converse** of $p \rightarrow q$ is $q \rightarrow p$
The **inverse** of $p \rightarrow q$ is $\sim p \rightarrow \sim q$
The **contrapositive** of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

- Examples from English

- Is an implication logically equivalent to its inverse, its converse, or its contrapositive? Let's check!

- Given an implication, what is the relationship between its converse and its inverse?

“If and Only If”

What is the meaning of “p if q”?
What is the meaning of “p only if q”?
What is the meaning of “**p if and only if q**”?

Biconditional Connective: **p $\leftrightarrow$ q**
“p if and only if q”

p	q	p $\leftrightarrow$ q
1	1	1
1	0	0
0	1	0
0	0	1

Necessary / Sufficient Conditions

"p implies q" is equivalent to saying either of these:

- p is a **sufficient condition** for q
- q is a **necessary condition** for p

"p if and only if q" is equivalent to saying:

- p is a **necessary and sufficient condition** for q

Arguments

An **argument** is a conjecture that says:

If you make certain assumptions, then a particular statement must follow.

- The assumptions are called **premises**
- The statement that (supposedly) follows is the **conclusion**

Example:

$$p \vee q$$
$$q \rightarrow r$$
$$\sim p$$
$$\overline{\therefore\ r}$$

Premises: $p \vee q$, $q \rightarrow r$, $\sim p$

Conclusion: $\therefore r$