Announcements

- Both homeworks (5A and 5B) are due next Wednesday. We will not be grading *every* question.

Another recurrence relation example

- Assume the following definition of a function:

$$a_0 = 1 \qquad a_1 = 2$$

$$(\forall k \in Z^{\geq 2})\left[a_k = a_{k-1} + a_{k-2}\right]$$

- Prove the following definition property, using strong induction:

$$(\forall n \in Z^{\geq 0})\left[a_n \leq 2^n\right]$$

Another example- a divisibility property

- Assume the following definition of a recurrence relation:

$$a_0 = 0$$

$$a_1 = 7$$

$$(\forall i \geq 2)\left[a_i = 2a_{i-1} + 3a_{i-2}\right]$$

- Prove using strong induction that all elements in this relation have this property:

$$(\forall n \in N)\left[a_n \equiv 0 \pmod{7}\right]$$

Yet another one…

- Assume the following definition of a recurrence relation:

$$a_0 = 0$$

$$a_1 = 4$$

$$(\forall i \geq 2)\left[a_i = 6a_{i-1} - 5a_{i-2}\right]$$

- Prove using strong induction that all elements in this relation have this property:

$$(\forall n \in N)\left[a_n = 5^n - 1\right]$$

Another example

- Theorem: for all $n \geq 2$: n can be expressed as the product of primes. (Note that we consider a single prime factor to be a “product” of primes.)

This is “half” of the Unique Prime Factorization Theorem. The other half would be to show that the prime factorization is *unique*.

Chocolate Bar Division

Suppose you have a chocolate bar that is sectioned off into n squares, arranged in a rectangle. You can break the bar into pieces along the lines separating the squares. (Each break must go all the way across the current piece.)

Claim: It will always take n-1 breaks to separate the bar into individual squares. (No matter how you proceed!)