

Announcements

- Homework #6 has been posted.

Recall Basic Definitions

Union: $x \in A \cup B \leftrightarrow x \in A \vee x \in B$

Intersection: $x \in A \cap B \leftrightarrow x \in A \wedge x \in B$

Complement: $x \in A^c \leftrightarrow x \notin A$

Difference: $x \in A - B \leftrightarrow x \in A \wedge x \notin B$

Proving Subset Relationship

Claim: $A \subseteq B$.

Proof:

Let $x \in A$.

...

$\therefore x \in B$.

Some Properties of Sets

- Inclusion $A \cap B \subseteq A$ $A \cap B \subseteq B$
 $A \subseteq A \cup B$ $B \subseteq A \cup B$
- Transitivity $A \subseteq B \wedge B \subseteq C \rightarrow A \subseteq C$
- Let's prove a few of these

Proving two sets are equal

Two (basic) techniques:

Claim: $A = B$.

Proof:

$x \in A \leftrightarrow$

$S1 \leftrightarrow$

$S2 \leftrightarrow$

$S3 \leftrightarrow$

...

$x \in B$

Claim: $A = B$.

Proof:

Part I. [Show $A \subseteq B$]

...

Part II. [Show $B \subseteq A$]

...

More Properties of Sets

- DeMorgan's for complement

$$(A \cup B)' = A' \cap B'$$

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- Distribution of union and intersection

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

- Let's prove a couple.
- There are a number of others as well; see the handout that is on the class webpage

Proving two sets are equal (Using the Rule Sheet)

A *third* way to prove two sets are equal:

Claim: $A = B$.

Proof:

$A = X$ [By rule called...]

$= Y$ [By rule called...]

$= Z$ [By rule called...]

$= B$ [By rule called...]

Deriving new properties
using rules (or from definitions)

$$B - (A \cap C) = (B - A) \cup (B - C)$$

$$A - B = A - (A \cap B)$$

$$\underline{A} \subseteq \underline{B} \wedge \underline{A} \subseteq \underline{C} \rightarrow \underline{A} \subseteq \underline{(B \cap C)}$$

Using Venn diagrams to help find counterexamples

$$A \cup (B \cap C) = ? = (A \cap B) \cup (A \cap C)$$

$$A \cup (B - C) = ? = (A \cup B) - C$$

Trick: Draw the Venn diagrams and find a cell where they disagree. Make sure your counterexample has an element in that cell.