Announcements

- Homework #8 due tomorrow.
- 2nd midterm on Thursday

Recall: Basic Probability Concepts

- Sample space = set of all possible outcomes
- Event = any subset of the sample space
- Classical formula (for Sample Space with equally likely elements)

$$P(E) = \frac{n(E)}{n(S)}$$

- Multiplication Rule:

  \# of ways to complete task = $n_1 * n_2 * \cdots * n_k$

  (Where n_i is the number of ways to complete step i)

Independent Events

Two events are said to be “independent” if knowledge about whether or not one of the event occurs does not effect the probability of the other event.

- Examples of Independent Events
- Examples of Events that are not Independent.

(Grade on midterms, seats on the bus, superbowl winner, etc.)

Multiplication Rule for Independent Events

Assume that events E_1, E_2, E_3… E_k are all independent. Then:

$$P(E_1 \cap E_2 \cap E_3 \cap \ldots \cap E_k) = P(E_1)P(E_2)P(E_3) \ldots P(E_k)$$

Multiplication Rule for Independent Events

- If you flip a coin 5 times, what is the probability that it will be heads every time?
- In Monopoly, you go to jail if you roll "doubles" three times in a row. What is the probability of this happening on a given turn?
- Steve Nash is the NBA player with the highest career free throw percentage, which is almost exactly 90%. If Steve went to the line 10 times, what is the probability that he would sink all ten free throws?

The Difference Rule

- If A is a finite set and B $\subseteq$ A, then

  n(A – B) = n(A) – n(B)

- One consequence:

$$n(E') = n(S) - n(E)$$

$$\frac{n(E')}{n(S)} = \frac{n(S)}{n(S)} - \frac{n(E)}{n(S)}$$

$$\boldsymbol{P(E') = 1 - P(E)}$$

Probabilities with Compliments

- What is the probability that your 4-digit PIN has at least one repeated digit?
- What is the probability that your Maryland license plate has at least one 7? (Guess first for fun!)
- A certain medication is 95% effective. (That means that if used properly for 1 year it will work 95% of the time.) What is the chance of at least one failure over a 10 year interval?

The Addition Rule

- If $A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_k = A$
- and $A_1, A_2, A_3, \ldots, A_k$ are **pairwise disjoint (mutually exclusive)**

In other words, if these subsets form a partition of A, then

$n(A) = n(A_1) + n(A_2) + n(A_3) + \cdots + n(A_k)$

and

$P(A) = P(A_1) + P(A_2) + P(A_3) + \cdots + P(A_k)$

Using the Addition Rule

- A group of 5 students are to be seated in 5 chairs. What is the probability that James ends up sitting next to Nancy? (Guess first!)
- What is the probability that James does NOT end up sitting next to Nancy? (Easy question…)
- If the group consists of 3 men and 2 women, what is the probability that all of the men will end up sitting next to each other? (Guess first!)

The inclusion/exclusion rule

Recall that the "Addition Rule" only works for events that are mutually exclusive. How can we calculate probabilities for unions of overlapping events?

Example:

The probability of the Orioles winning the World Series this year is 30%

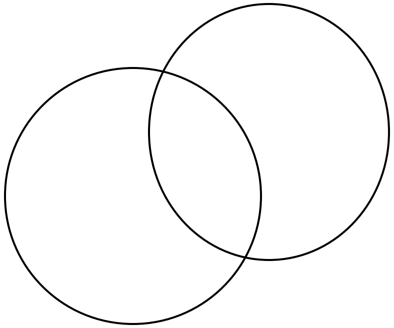
The probability of the Redskins winning the Super Bowl this year is 10%.

What is the probability that at least one of these events occurs?

The inclusion/exclusion rule

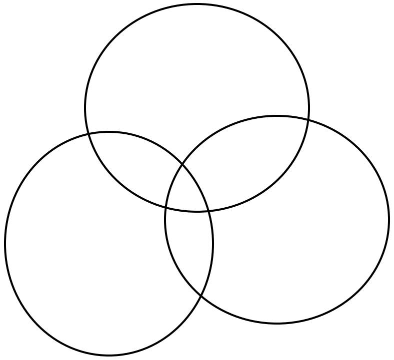
- If there are two sets:

  $n(A \cup B) = n(A) + n(B)$
  $- n(A \cap B)$



- If there are three sets:

  $n(A \cup B \cup C) =$
  $n(A) + n(B) + n(C)$
  $- n(A \cap B) - n(A \cap C) - n(B \cap C)$
  $+ n(A \cap B \cap C)$



Decision Tree

- People= {Alice, Bob, Carolyn, Dan}
- Need to be appointed as president, vice-president, and treasurer, and nobody can hold more than one office
  - how many ways can it be done with no restrictions?
  - how many ways can it be done if Alice doesn't want to be president?
  - how many ways can it be done if Alice doesn't want to be president, and only Bob and Dan are willing to be vice-president?

Probability Tree

- Depicts scenario that happens in stages
- Makes it easy to answer almost any probability question about the outcome

Example:

John and Sarah are playing a chess tournament. They will play the best two out of three games.

- Sarah has a slight edge, so she has a 60% chance of winning the first game.
- If Sarah wins the first game, she gains confidence, so her chance of winning the second game is 70%
- If Sarah loses the first game, she loses confidence, so her chance of winning the second game is 50%
- The third game (if there is one) is back to 60% chance for Sarah.

Questions:

- What is the probability that Sarah wins the tournament?
- What is the probability that the tournament ends in two games?
- What is the probability that John wins, but it takes three games?