

# Announcements

- The final exam is on 5/16 (Saturday) at 4:00PM.

<b>Sections</b>	<b>Exam Room</b>
0101, 0102, 0103, 0104	HJP 0226
0201, 0202, 0203	MMH 1400
0301, 0302, 0303, 0304	PHY 1412

# Recall: Properties of binary relations

Reflexive  $(\forall a \in A) [aRa]$

Irreflexive  $(\forall a \in A) [a \not R a]$

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Symmetric  $(\forall a, b \in A) [aRb \rightarrow bRa]$

Antisymmetric  $(\forall a, b \in A) [aRb \wedge bRa \rightarrow a = b]$

Asymmetric  $(\forall a, b \in A) [aRb \rightarrow b \not R a]$

Non-symmetric  $(\forall a, b \in A) [a \neq b \rightarrow (aRb \leftrightarrow b \not R a)]$

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Transitive  $(\forall a, b, c \in A) [aRb \wedge bRc \rightarrow aRc]$

# Equivalence relations

- A binary relation is an **equivalence relation** iff it is:
  - reflexive,
  - symmetric, and
  - Transitive

## Example:

Let  $R$  be the relation over  $\mathbb{Z} \times \mathbb{Z}$  defined by:

$$aRb \text{ iff } a \equiv_4 b$$

(Let's verify that this is an equivalence relation.)

# Equivalence relations

- An equivalence relation forms a *partition* of the elements:  
All elements that are related to one another are within the same partition.
- These partitions are called equivalence classes
  - $[a]$  = the equivalence class containing  $a$
  - $[a] = \{x \in A \mid xRa\}$

# More Equivalence Relations

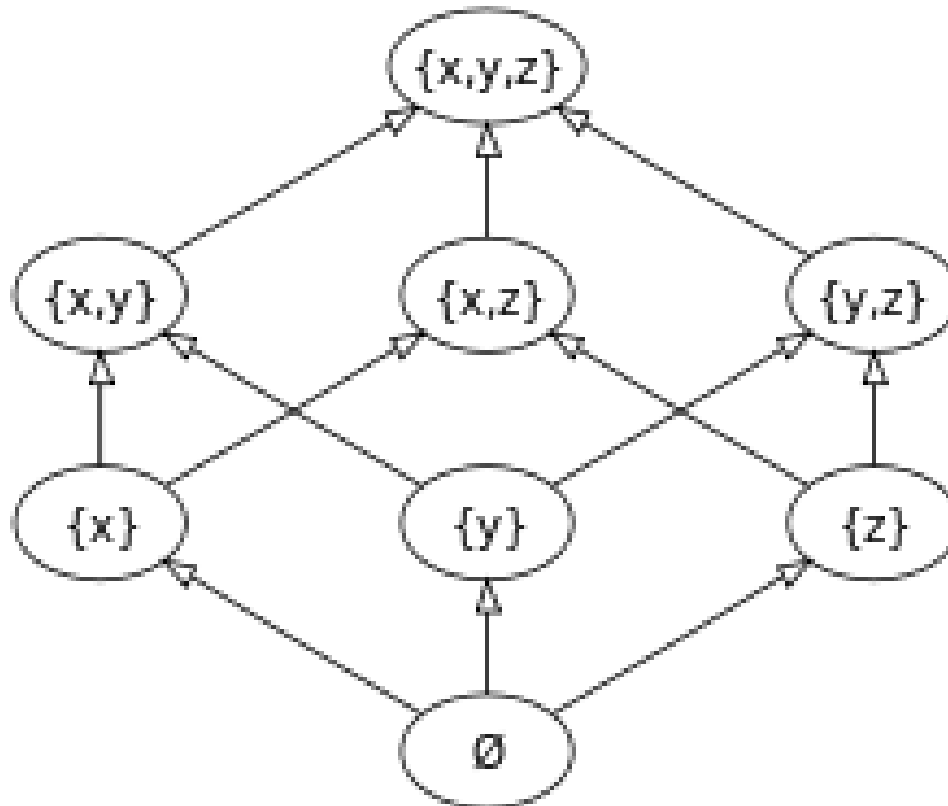
- Let  $X = \{a,b,c,d,e,f\}$ , and define the following binary relation over  $X$ :  
$$R = \{(a,a),(b,b),(c,c),(d,d),(e,e),(f,f),(a,e),(a,d),(d,a),(d,e),$$
$$(e,a),(e,d),(b,f),(f,b)\}$$
- Let  $R$  be a binary relation defined over the 50 states in the U.S. as:  
 $aRb$  iff the names of  $a$  and  $b$  start with the same letter
- Let  $R$  be a binary relation over  $\mathbb{R}$  defined by:  
 $aRb$  iff  $\sin(a) = \sin(b)$
- Let  $f$  be *any* function with domain  $D$ . Define a binary relation  $R$  over  $D$  as:  
 $aRb$  iff  $f(a)=f(b)$

# Partial order relation

- R is a **partial order relation** if and only if R is reflexive, antisymmetric, and transitive
- Examples
  - $\geq$  over  $\mathbb{Z}$
  - divisibility over  $\mathbb{Z}^+$
  - $\subseteq$  over any collection of sets

# Partial order relation

- Partial orders correspond to “reachability” in *directed acyclic graphs* (DAGs)



# Total ordering

A relation,  $R$ , is **total** (over  $S$ ) if for all elements  $a, b \in S$ :  
 $aRb$  or  $bRa$

A relation is a **total order** relation if it is:

- Total
- Transitive
- Antisymmetric

## Examples:

- $\leq$  over  $\mathbb{R}$
- Lexicographical ordering of English words