# CMSC 330: Organization of Programming Languages

## Regular Expressions and Finite Automata



## Introduction

- That's it for the basics of Ruby
  - If you need other material for your project, come to office hours or check out the documentation
- Next up: How do regular expressions (REs) really work?
  - Mixture of a very practical tool (string matching with REs) and some nice theory
  - A great computer science result



## A Few Questions About REs

- What does a regular expression represent?
  - Just a set of strings
- What are the basic components of REs?
  - E.g., we saw that e+ is the same as ee*
- How are REs implemented?
  - We'll see how to build a structure to parse REs



## Definition: Alphabet

- An alphabet is a finite set of symbols
  - Usually denoted Σ
- Example alphabets:
  - Binary: Σ = {0,1}
  - Decimal: Σ = {0,1,2,3,4,5,6,7,8,9}
  - Alphanumeric: Σ = {0-9,a-z,A-Z}

## Definition: String

- A string is a finite sequence of symbols from $\Sigma$
  - $\varepsilon$ is the empty string ("" in Ruby)
  - $|s|$ is the length of string s
    - $|\text{Hello}| = 5$, $|\varepsilon| = 0$
  - Note
    - $\varnothing$ is the empty set (with 0 elements)
    - $\varnothing \neq \{ \varepsilon \} \neq \varepsilon$
- Example strings:
  - $0101 \in \Sigma = \{0,1\}$ (binary)
  - $0101 \in \Sigma =$ decimal
  - $0101 \in \Sigma =$ alphanumeric



## Definition: String concatenation

- String concatenation is indicated by juxtaposition
  - If $s_1$ = super and $s_2$ = hero, then $s_1s_2$ = superhero
  - Sometimes also written $s_1 \cdot s_2$
  - For any string s, we have $s\varepsilon = \varepsilon s = s$
  - You can concatenate strings from different alphabets; then the new alphabet is the union of the originals:
    - If $s_1$ = super $\in \Sigma_1 = \{s,u,p,e,r\}$ and $s_2$ = hero $\in \Sigma_2 = \{h,e,r,o\}$, then $s_1s_2$ = superhero $\in \Sigma_3 = \{e,h,o,p,r,s,u\}$



## Definition: Language

- A language L is a set of strings over an alphabet
- Example: The set of phone numbers over the alphabet $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 9, (, ), -\}$
  - Give an example element of this language `(123) 456-7890`
  - Are all strings over the alphabet in the language? `No`
  - Is there a Ruby regular expression for this language?

```
/\(\d{3,3}\) \d{3,3}-\d{4,4}/
```

- Example: The set of all strings over $\Sigma$
  - Often written $\Sigma^*$



## Definition: Language (cont.)

- Example: The set of strings of length 0 over the alphabet $\Sigma = \{a, b, c\}$
  - $L = \{ s \mid s \in \Sigma^* \text{ and } |s| = 0\} = \{\varepsilon\} \neq \varnothing$
- Example: The set of all valid Ruby programs
  - Is there a Ruby regular expression for this language?

No. Matching (an arbitrary number of) brackets so that they are balanced is impossible using REs { { { … } } }

- Can REs represent all possible languages?
  - The answer turns out to be no!
  - The languages represented by regular expressions are called, appropriately, the regular languages

## Operations on Languages

- Let $\Sigma$ be an alphabet and let $L$, $L_1$, $L_2$ be languages over $\Sigma$
- Concatenation $L_1L_2$ is defined as
  - $L_1L_2 = \{xy \mid x \in L_1 \text{ and } y \in L_2\}$
- Union is defined as
  - $L_1 \cup L_2 = \{ x \mid x \in L_1 \text{ or } x \in L_2\}$
- Kleene closure is defined as
  - $L^* = \{ x \mid \varepsilon \text{ or } x \in L \text{ or } x \in LL \text{ or } x \in LLL \text{ or } \ldots\}$



## Definition: Regular Expressions

- Given an alphabet $\Sigma$, the regular expressions over $\Sigma$ are defined inductively as

| regular expression | denotes language |
|---|---|
| $\emptyset$ | $\emptyset$ |
| $\varepsilon$ | $\{\varepsilon\}$ |
| each element $\sigma \in \Sigma$ | $\{\sigma\}$ |

**Constants**



## Definition: Regular Expressions (cont.)

- Let A and B be regular expressions denoting languages $L_A$ and $L_B$, respectively

| regular expression | denotes language |
|---|---|
| AB | $L_AL_B$ |
| (A\|B) | $L_A \cup L_B$ |
| A* | $L_A^*$ |

**Operations**

- There are no other regular expressions over $\Sigma$



## Regular Expressions Denote Languages

- By applying operations on constants
  - Generates a set of strings (i.e., a language)
  - Examples
    - a → {“a”}
    - a|b → {“a”} ∪ {“b”} = {“a”, “b”}
    - a* → {ε} ∪ {“a”} ∪ {“aa”} ∪ … = {ε,“a”, “aa”, … }
- If s ∈ language generated by a RE r, we say that r accepts, describes, or recognizes string s

## Precedence

- Order in which operators are applied
  - In arithmetic
    - Multiplication × > addition +
    - 2 × 3 + 4 = (2 × 3) + 4 = 10
  - In regular expressions
    - Kleene closure * > concatenation > union |
    - ab|c = ( a b ) | c = {"ab", "c"}
    - ab* = a ( b* ) = {"a", "ab", "abb"…}
    - a|b* = a | ( b* ) = {"a", "", "b", "bb", "bbb"…}
  - Can change order using parentheses ( )
    - E.g., a(b|c), (ab)*, (a|b)*



## Regular Languages

- The languages that can be described using regular expressions are the regular languages or regular sets
- Not all languages are regular
  - Examples (without proof):
    - The set of palindromes over Σ
      - reads the same backward or forward
    - $\{a^nb^n \mid n > 0\}$ ($a^n$ = sequence of n a's)
- Almost all programming languages are not regular
  - But aspects of them sometimes are (e.g., identifiers)
  - Regular expressions are commonly used in parsing tools



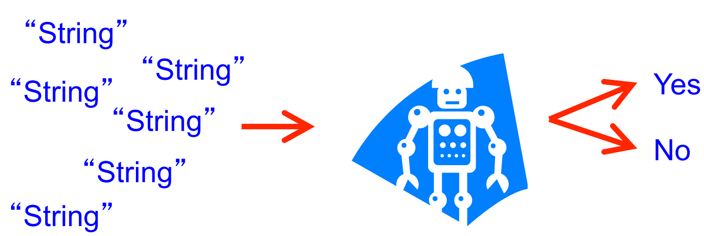
## Ruby Regular Expressions

- Almost all of the features we've seen for Ruby REs can be reduced to this formal definition
  - /Ruby/ – concatenation of single-character REs
  - /(Ruby|Regular)/ – union
  - /(Ruby)*/ – Kleene closure
  - /(Ruby)+/ – same as (Ruby)(Ruby)*
  - /(Ruby)?/ – same as (ε|(Ruby)) (// is ε)
  - /[a-z]/ – same as (a|b|c|...|z)
  - / [^0-9]/ – same as (a|b|c|...) for a,b,c,... ∈ Σ - {0..9}
  - ^, $ – correspond to extra characters in alphabet

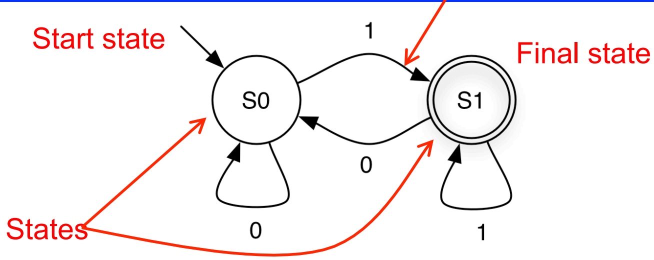


## Implementing Regular Expressions

- We can implement a regular expression by turning it into a finite automaton
  - A "machine" for recognizing a regular language



## Finite Automata

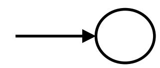



- Machine starts in start or initial state
- Repeat until the end of the string is reached
  - Scan the next symbol s of the string
  - Take transition edge labeled with s
- String is accepted if automaton is in final state when end of string reached



## Finite Automata: States

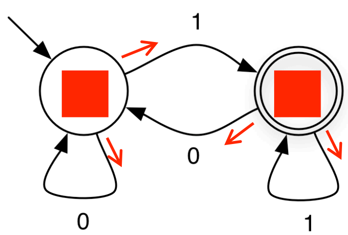
- Start state
  - State with incoming transition from no other state
  - Can have only 1 start state



- Final states
  - States with double circle
  - Can have 0 or more final states
  - Any state, including the start state, can be final




## Finite Automaton: Example 1

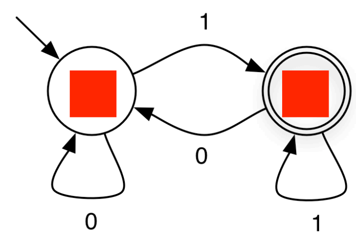



0 0 1 0 1 1

accepted



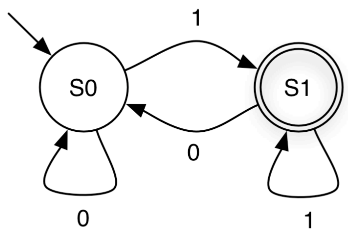
## Finite Automaton: Example 2




0 0 1 0 1 0

not accepted

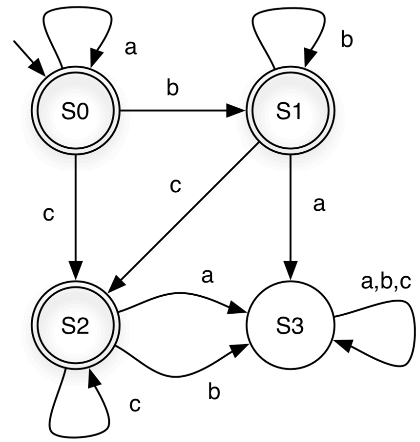
## What Language is This?




- All strings over {0, 1} that end in 1
- What is a regular expression for this language?
  - (0|1)*1



## Finite Automaton: Example 3

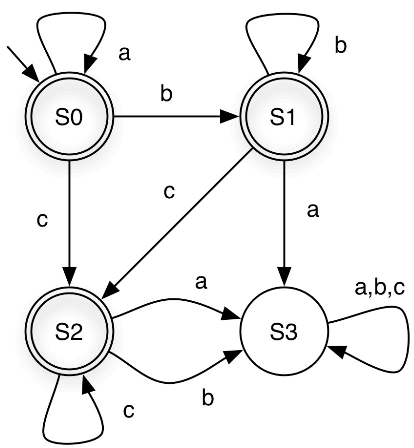



| string | state at end | accepts ? |
|---:|---:|---:|
| aabcc | S2 | Y |
| acc | S2 | Y |
| bbc | S2 | Y |
| aabbb | S1 | Y |
| aa | S0 | Y |
| ε | S0 | Y |
| acba | S3 | N |

(a,b,c notation shorthand for three self loops)



## Finite Automaton: Example 3 (cont.)




What language does this DFA accept?

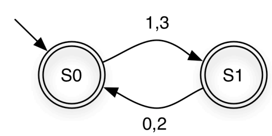
a*b*c*

S3 is a dead state – a nonfinal state with no transition to another state

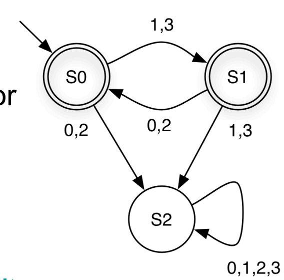


## Dead State: Shorthand Notation

- If a transition is omitted, assume it goes to a dead state that is not shown

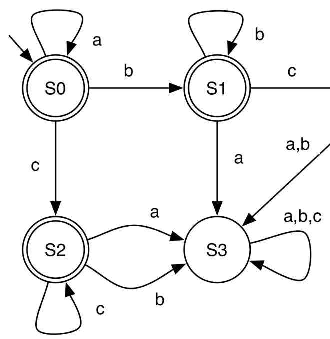



is short for




- Language?
  - Strings over {0,1,2,3} with alternating even and odd digits, beginning with odd digit

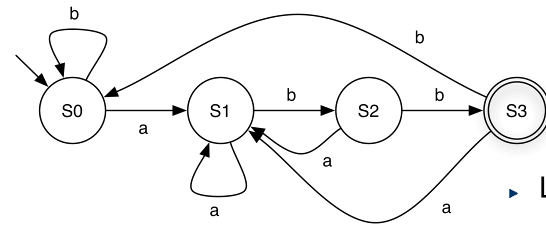
# Finite Automaton: Example 4




a*b*c* again, so DFAs are not unique



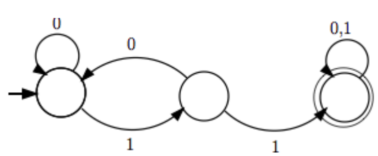
# Finite Automaton: Example 5




- Language?
  - (a|b)*abb
- Description for each state
  - S0 = "Haven't seen anything yet" OR "seen zero or more b's" OR "Last symbol seen was a b"
  - S1 = "Last symbol seen was an a"
  - S2 = "Last two symbols seen were ab"
  - S3 = "Last three symbols seen were abb"

# Practice

Give the English descriptions and the DFA or regular expression of the following languages:

- ((0|1)(0|1)(0|1)(0|1)(0|1))*
  - All strings with length a multiple of 5
- (01)*|(10)*|(01)*0|(10)*1
  - All alternating binary strings
- 


  All binary strings containing the substring "11"



# Practice

- Give the regular expressions and finite automata for the following languages
  - You and your neighbors' names
  - All protein-coding DNA strings (including only ATCG and appearing in multiples of 3)
  - All binary strings containing an even length substring of all 1's
  - All binary strings containing exactly two 1's
  - All binary strings that start and end with the same number

# Review

- Languages
  - Sets of strings
  - Operations on languages
- Regular expressions
  - Constants
  - Operators
  - Precedence
- Finite automata
  - States
  - Transitions
  - Accept strings