Periodic Scheduling

We have a periodic scheduling problem. There are n tasks. Each task takes one unit of time to perform. The requirement is that task i should be scheduled once in each time period p_i.

For example if task i, has period p_i. We must schedule it in each time-window of the form $[p_i \cdot j + 1, \ldots, p_i \cdot (j+1)]$ for $j = 0....$ Let $L = lcm(p_1, p_2, \ldots, p_n)$. Moreover suppose that $\sum_i \frac{1}{p_i} \leq 1$. Show how to find a periodic schedule for the first L time slots. If the above condition is not satisfied, there is no periodic schedule. Why?

We create a network flow instance as follows. We create a set of vertices T, such that corresponding to each task i that we have to schedule, there are $\frac{L}{p_i}$ nodes t_i^j for $j = 1 \ldots \frac{L}{p_i}$. There is a source s as well. There are unit capacity edges from s to the nodes in T. Note that we have to schedule task i, $\frac{L}{p_i}$ times. We have a set of U vertices, one for each of the L time slots. There are unit capacity edges from each node in L to the sink t. For each task i, we have p_i edges going from t_i^j to $j \cdot p_i + k$ for $k = 0 \ldots p_i - 1$. This means that the j^{th} instance of task i has to be scheduled in time slot $j \cdot p_i + k$ for some $0 \geq k < p_i$. If this network has a flow of value $\sum_i \frac{L}{p_i}$ then a feasible solution exists. Note that if the required condition is satisfied then the cut corresponding to the edges out of s is a min cut. To see this, we demonstrate a flow of this value. Saturate all the edges out of s. Each t_i^j node sends $\frac{1}{p_i}$ units of flow to each of its p_i outgoing edges. Because of the required condition on p_i's, the total flow into a node of U is at most 1. This is a feasible flow.

Note that the condition on the p_i's is a necessary condition. Assume that a feasible schedule exists then each task i is scheduled $\frac{L}{p_i}$ times. Summing over all i gives the total number of scheduled slots. This implies that $\sum_i \frac{L}{p_i} \leq L$, which implies the condition.