CMSC451 Spring 2015 Final Review Session

May 9, 2015

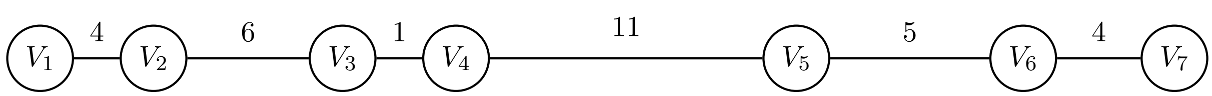
1. (Easy.)

   a Define the class **P**.

   b Define the class **NP**.

2. (Easy-Medium.) In the SET COVER problem, you are given a set $\mathcal{U}$ of ground elements $e_1, e_2, \ldots, e_n$. You are also given a set $\mathcal{S}$ consisting of m subsets $S_1, S_2, \ldots, S_m$ of $\mathcal{U}$. Given a positive integer k, is it possible to select at most k subsets from $\mathcal{S}$ so that every element e_i from $i = 1, \ldots, n$ is contained in at least one selected subset? I.e., is there a set of at most k subsets whose union covers $\mathcal{U}$? Prove that SET COVER is NP-Complete. You may want to consider giving a reduction from VERTEX COVER.

3. (Easy-Medium, from Quiz 2.) Suppose you want to hike from village V_1 to village V_N in K days. There are a number of intermediate villages along the way where you can stay overnight. Let $d(i, i+1)$ denote the distance between villages V_i and V_{i+1}. The goal is to minimize the maximum amount of walking you have to do in a single day.

   For example, consider the situation in which you need to go from V_1 to V_7 in 3 days (see figure). If you stop at V_3 and V_5, then on day 1, you walk from V_1 to V_3 (10 miles), on day 2 you walk from V_3 to V_5 (12 miles) and on day 3, you walk from V_5 to V_7 (9 miles). Then the maximum distance walked per day is 12 miles. A better solution is to walk from V_1 to V_4 on the first day (11 miles), from V_4 to V_5 on day 2 (11 miles) and V_5 to V_7 on day 3 (9 miles): the maximum distance walked per day is only 11 miles.

   Design a dynamic programming algorithm to minimize the maximum amount of walking in a single day. No proof required, but give time complexity and explain your recurrence.

4. (Medium-Hard.) Say you have access to a function DICT that returns true if its input is a valid English word, and false otherwise. We are given as input a sentence from which the punctuation has been stripped (for example, "dynamicprogrammingisfabulous"). Assuming calls to DICT take constant time, give an $O(n^2)$ time algorithm to determine whether an input string of length n can be split into a sequence of valid words.

5. (Medium.) In the famous TRAVELING SALESMAN PROBLEM (TSP), a salesman must visit n cities labeled $v_1, v_2, \ldots, v_n$. The salesman starts in city v_1, his home, and wants to find a tour, i.e., an order in which to visit all the other cities and return home. Formally, given a set of distances of n cities, and a bound D, is there a tour of length at most D? Distances are non-negative and need not be symmetric. Prove that TSP is NP-complete.

6. (Hard.) (Adapted from KT, p. 412) Suppose you have n doctors at a hospital who need to collectively cover the vacation days over the next year. There are k vacation periods, each spanning several contiguous days. Let D_j be the set of days included in the jth vacation period. Each doctor i has a set S_i of days when he or she can work; these availabilities need not form a contiguous time period, even within a single vacation period. For a fixed c, each doctor should be assigned to work at most c vacation days total, and only on days when he or she is available. Also, for each vacation period j, each doctor should be assigned to work at most one of the days in the set D_j. Give a polynomial-time algorithm that determines whether it is possible to validly assign a single doctor to each vacation day and still cover every vacation day.