

# CMSC 330: Organization of Programming Languages

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## OCaml Higher Order Functions

# Anonymous Functions

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- ▶ Recall code blocks in Ruby

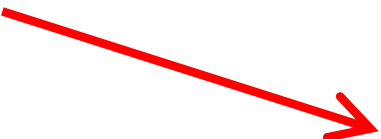

```
(1..10).each { |x| print x }
```

- Here, we can think of `{ |x| print x }` as a function
- ▶ We can do this (and more) in OCaml

# Anonymous Functions

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- ▶ Passing functions around is very common
  - So often we don't want to bother to give them names
- ▶ Use **fun** to make a function with no name

Parameter  Body 

```
fun x -> x + 3
```

**(fun x -> x + 3) 5 = 8**

# Anonymous Functions

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## ▶ Syntax

- **fun**  $x_1$  ...  $x_n$   $\rightarrow$   $e$

## ▶ Evaluation

- An anonymous function is an expression
- In fact, *it is a value* – no further evaluation is possible
  - As such, it can be passed to other functions, returned from them, stored in a variable, etc.

## ▶ Type checking

- **(fun**  $x_1$  ...  $x_n$   $\rightarrow$   $e$  **)** : **(** $t_1$   $\rightarrow$  ...  $\rightarrow$   $t_n$   $\rightarrow$   $u$ **)**  
if  $e : u$  under assumptions  $x_1 : t_1$ , ...,  $x_n : t_n$ .
  - (Same rule as **let**  $f$   $x_1$  ...  $x_n = e$ )

# All Functions Are Anonymous

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- ▶ Functions are **first-class**, so you can bind them to other names as you like

```
let f x = x + 3;;
```

```
let g = f;;
```

```
g 5 = 8
```

- ▶ In fact, **let** for functions is syntactic **shorthand**

```
let f x = body
```



is semantically equivalent to

```
let f = fun x -> body
```



# Example Shorthands

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- ▶ `let next x = x + 1`
  - Short for `let next = fun x -> x + 1`
- ▶ `let plus x y = x + y`
  - Short for `let plus = fun x y -> x + y`
- ▶ `let rec fact n =`  
    `if n = 0 then 1 else n * fact (n-1)`
  - Short for `let rec fact = fun n ->`  
    `(if n = 0 then 1 else n * fact (n-1))`

# Defining Functions Everywhere

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```
let move l x =  
  let left x = x - 1 in (* locally defined fun *)  
  let right x = x + 1 in (* locally defined fun *)  
  if l then left x  
  else      right x  
;;
```

```
let move' l x = (* equivalent to the above *)  
  if l then (fun y -> y - 1) x  
  else      (fun y -> y + 1) x
```

# Calling Functions, Generalized

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- ▶ Syntax  $e_0 e_1 \dots e_n$  *Not just a variable  $f$*
- ▶ Evaluation
- Evaluate arguments  $e_1 \dots e_n$  to values  $v_1 \dots v_n$ 
    - Order is actually right to left, not left to right
    - But this doesn't matter if  $e_1 \dots e_n$  don't have side effects
  - Evaluate  $e_0$  to a function **fun**  $x_1 \dots x_n \rightarrow e$
  - Substitute  $v_i$  for  $x_i$  in  $e$ , yielding new expression  $e'$
  - Evaluate  $e'$  to value  $v$ , which is the final result



# Calling Functions, Generalized

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- ▶ Syntax  $e_0 e_1 \dots e_n$
- ▶ Type checking (almost the same as before)
  - If  $e_0 : t_1 \rightarrow \dots \rightarrow t_n \rightarrow u$  and  $e_1 : t_1, \dots, e_n : t_n$  then  $e_0 e_1 \dots e_n : u$
- ▶ Example:
  - $(\text{fun } x \rightarrow x+1) 1 : \text{int}$
  - since  $(\text{fun } x \rightarrow x+1) : \text{int} \rightarrow \text{int}$  and  $1 : \text{int}$

# Pattern Matching With Fun

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- ▶ `match` can be used within `fun`

```
(fun l -> match l with (h::_) -> h) [1; 2]  
= 1
```

- ▶ But use named functions for complicated matches
- ▶ May use standard pattern matching abbreviations

```
(fun (x, y) -> x+y) (1, 2)  
= 3
```

# Passing Functions as Arguments

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- ▶ In OCaml you can pass functions as arguments (akin to Ruby code blocks)

```
let plus_three x = x + 3 (* int -> int *)
```

```
let twice f z = f (f z) (* ('a->'a) -> 'a -> 'a *)
```

```
twice plus_three 5 = 11
```

- ▶ Ruby's `collect` is called `map` in OCaml
  - `map f l` applies function `f` to each element of `l`, and puts the results in a new list (preserving order)

```
map plus_three [1; 2; 3] = [4; 5; 6]
```

```
map (fun x -> (-x)) [1; 2; 3] = [-1; -2; -3]
```

# The Map Function

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- ▶ Let's write the `map` function
  - Takes a function and a list, applies the function to each element of the list, and returns a list of the results

```
let rec map f l = match l with
  [] -> []
  | (h::t) -> (f h)::(map f t)
```

```
let add_one x = x + 1
```

```
let negate x = -x
```

```
map add_one [1; 2; 3] = [2; 3; 4]
```

```
map negate [9; -5; 0] = [-9; 5; 0]
```

- ▶ Type of `map`?

# The Map Function (cont.)

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- ▶ What is the type of the map function?

```
let rec map f l = match l with  
  [] -> []  
  | (h::t) -> (f h) :: (map f t)
```

$(\text{'a} \rightarrow \text{'b}) \rightarrow \text{'a list} \rightarrow \text{'b list}$

The diagram illustrates the type signature of the map function. It shows the expression  $(\text{'a} \rightarrow \text{'b}) \rightarrow \text{'a list} \rightarrow \text{'b list}$ . A red bracket is drawn under the first part,  $(\text{'a} \rightarrow \text{'b})$ , with the letter  $f$  centered below it. Another red bracket is drawn under the second part,  $\text{'a list} \rightarrow \text{'b list}$ , with the letter  $l$  centered below it.

This is the `fold_left` function in OCaml's standard `List` library

# The Fold Function

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## ▶ Common pattern

- Iterate through list and apply function to each element, keeping track of partial results computed so far

```
let rec fold f a l = match l with
  [] -> a
  | (h::t) -> fold f (f a h) t
```

- `a` = “accumulator”
- Usually called `fold left` to remind us that `f` takes the accumulator as its first argument

## ▶ What's the type of `fold`?

= `('a -> 'b -> 'a) -> 'a -> 'b list -> 'a`

# Example

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```
let rec fold f a l = match l with
  [] -> a
  | (h::t) -> fold f (f a h) t
```

```
let add a x = a + x
fold add 0 [1; 2; 3; 4] →
fold add 1 [2; 3; 4] →
fold add 3 [3; 4] →
fold add 6 [4] →
fold add 10 [] →
10
```

We just built the `sum` function!

# Another Example

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```
let rec fold f a l = match l with
  [] -> a
  | (h::t) -> fold f (f a h) t
```

```
let next a _ = a + 1
fold next 0 [2; 3; 4; 5] →
fold next 1 [3; 4; 5] →
fold next 2 [4; 5] →
fold next 3 [5] →
fold next 4 [] →
4
```

We just built the `length` function!



# Using Fold to Build Reverse

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```
let rec fold f a l = match l with
  [] -> a
  | (h::t) -> fold f (f a h) t
```

- ▶ Can you build the **reverse** function with **fold**?

```
let prepend a x = x::a
```

```
fold prepend [] [1; 2; 3; 4] →
```

```
fold prepend [1] [2; 3; 4] →
```

```
fold prepend [2; 1] [3; 4] →
```

```
fold prepend [3; 2; 1] [4] →
```

```
fold prepend [4; 3; 2; 1] [] →
```

```
[4; 3; 2; 1]
```

# Quiz 1: What does this evaluate to?

---

```
let id x = x in
  (fun f y -> f (y+1)) id 1
```

- A. Error
- B. 2
- C. 1
- D. (id 2)

# Quiz 1: What does this evaluate to?

---

```
let id x = x in  
  (fun f y -> f (y+1)) id 1
```

A. Error

B. 2

C. 1

D. (id 2)

## Quiz 2: What does this evaluate to?

---

```
map (fun x -> x *. 4) [1;2;3]
```

- A. [ 1.0; 2.0; 3.0 ]
- B. [ 4.0; 8.0; 12.0 ]
- C. Error
- D. [4; 8; 12 ]

## Quiz 2: What does this evaluate to?

---

```
map (fun x -> x *. 4) [1;2;3]
```

A. [ 1.0; 2.0; 3.0 ]

B. [ 4.0; 8.0; 12.0 ]

**C. Error**

D. [4; 8; 12 ]

## Quiz 3: What does this evaluate to?

---

```
fold (fun a y -> y::a) [] [2;3;4]
```

- A. [ 9 ]
- B. [ 2;5;9 ]
- C. [ 4;3;2 ]
- D. Error

## Quiz 3: What does this evaluate to?

---

```
fold (fun a y -> y::a) [] [2;3;4]
```

- A. [ 9 ]
- B. [ 2;5;9 ]
- C. [ 4;3;2 ]
- D. Error

## Quiz 4: What does this evaluate to?

---

```
let is_even x = (x mod 2 = 0) in  
map is_even [1;2;3;4;5]
```

- A. `[false;true;false;true;false]`
- B. `[0;1;1;2;2]`
- C. `[0;0;0;0;0]`
- D. `false`



## Quiz 4: What does this evaluate to?

---

```
let is_even x = (x mod 2 = 0) in  
map is_even [1;2;3;4;5]
```

- A. **[false;true;false;true;false]**
- B. [0;1;1;2;2]
- C. [0;0;0;0;0]
- D. false