1.
   (a) Show that you can find the median of three elements with three comparisons steps.
   (b) Show that you cannot find the median of three elements with fewer than three comparisons steps (in the worst case). Make your proof brief.

2. Assume that you use the selection algorithm from class (and from CLRS) but we use columns of size 3. We will show that the number of comparisons (in the worst case) is $O(n \log n)$. You can assume that n is "nice".
   (a) Briefly list each step of the algorithm and how many comparisons the step takes.
   (b) Write a recurrence for the number of comparisons the algorithm uses.
   (c) Solve the recurrence using constructive induction. Just get the high order term exactly.
   (d) You could also perform selection by simply doing Merge Sort, which asymptotically does $n \lg n$ comparisons. How does the number of comparisons this algorithm uses compare to using Merge Sort for selection?

3. Again, assume that you use the selection algorithm from class (and from CLRS) with columns of size 3. We will argue that the average number of comparisons is $O(n)$. Assume that the pivot is equally likely to be in any *legitimate* position. Assume (pessimistically) that the recursive call always works on the larger side (as done in class and in CLRS). You can assume that n is "nice".
   (a) Write a recurrence for (an upper bound on) the average number of comparisons the algorithm uses.
   (b) Solve the recurrence using constructive induction. Just get the high order term exactly.
   (c) How does this compare to the $4n$ upper bound on the randomized selection algorithm?