# Midterm 1

## CMSC 430
### Introduction to Compilers
### Fall 2013

October 16, 2013

## Instructions

**This exam contains 9 pages, including this one. Make sure you have all the pages. Write your name on the top of this page before starting the exam.**

Write your answers on the exam sheets. If you finish at least 15 minutes early, bring your exam to the front when you are finished; otherwise, wait until the end of the exam to turn it in. Please be as quiet as possible.

If you have a question, raise your hand. If you feel an exam question assumes something that is not written, write it down on your exam sheet. Barring some unforeseen error on the exam, however, you shouldn't need to do this at all, so be careful when making assumptions.

| Question | Score | Max |
|---|---|---|
| 1 | | 20 |
| 2 | | 40 |
| 3 | | 15 |
| 4 | | 25 |
| Total | | 100 |

**Question 1. Short Answer (20 points).**

**a. (5 points)** Briefly explain the difference between *bottom-up* and *top-down* parsing.

> **Answer:** A bottom-up parser constructs the parse tree starting from the leaves and working its way to the root. As a result, bottom-up parsers use productions in "reverse," as reductions from the right-hand side to the left-hand side. A top-down parser starts from the root of the parse tree and works its way down to the leaves. Top-down parsers use productions in the traditional way, replacing left-hand side nonterminals with the right-hand side.

**b. (5 points)** In at most a few sentences, explain what it means for a term to be *stuck*.

> **Answer:** A term is *stuck* if it is not fully reduced (i.e., not a value) and yet it is impossible to reduce it more.

**c. (5 points)** Briefly explain what the *preservation theorem* is.

>**Answer:** The preservation theorem says that if a term is well-typed and it is evaluated one step, then the resulting term has the same type. More precisely, we showed *If* $\cdot \vdash e : t$ *and* $e \rightarrow e'$ *then* $\cdot \vdash e' : t$.

**d. (5 points)** Here is a simplified version of the `json` type from `Yojson.Safe`:

```
type json = [ `Assoc of (string * json) list
       | `Int of int
       | `List of json list ]
```

Write an OCaml function that returns the sum of all the `` `Int `` elements of the type `json` above. For example, for `` `List [`Int 2; `Assoc ["foo'', `Int 3]] `` it should return 5. You may use any standard OCaml library functions you like, including the following two:

```
# List.fold_left;;
- : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a = <fun>
# List.map;;
- : ('a -> 'b) -> 'a list -> 'b list = <fun>
```

>**Answer:**
>
>```
>let rec sum = function
>| `Assoc l -> sum (`List (List.map snd l))
>| `Int i -> i
>| `List l -> List.fold_left (fun a b -> a + (sum b)) 0 l
>```

3

**Question 2. Parsing (40 points).**

**a. (25 points)** Consider the following ocamlyacc output:

```
0  $accept : %entry% $end
1  s : s a B
2    | C
3  a :
4    | A A
5  %entry% : '\001' s

state 0
      $accept : . %entry% $end (0)

      '\001'  shift  1
      .   error
      %entry% goto 2

state 1
      %entry% : '\001' . s  (5)

      C  shift  3
      .   error
      s  goto 4
```

```
state 2
      $accept : %entry% . $end (0)

      $end  accept

state 3
      s : C .   (2)

      .   reduce 2

state 4
      s : s . a B  (1)
      %entry% : '\001' s .   (5)
      a : .   (3)

      A  shift  5
      $end  reduce 5
      B  reduce 3
      a  goto 6
```

```
state 5
      a : A . A  (4)

      A  shift  7
      .   error

vstate 6
      s : s a . B  (1)

      B  shift  8
      .   error

state 7
      a : A A .   (4)

      .   reduce 4

state 8
      s : s a B .   (1)

      .   reduce 1
```

**i. (5 points)** What are the terminals of this grammar, and what are the nonterminals? Exclude the extra symbols added by ocamlyacc.

   **Answer:** The terminals are A, B, and C. The non-terminals are s and a.

**ii. (15 points)** Consider the following table showing the sequence of steps taken by the above parser. Fill in the missing entries in the input and stack columns. Note that the parse starts in state 1, and that the parse ends upon reduction r1. We've put the $end terminal on the end of the input for you.

| Stack | Input | Action |
|---|---|---|
| 1 [nothing else to fill in] | [fill in]                 $CAAB\$end$ | $s3$ |
| $1, C, 3$ | $AAB\$end$ | $r2$ |
| $1, s, 4$ | $AAB\$end$ | $s5$ |
| $1, s, 4, A, 5$ | $AB\$end$ | $s7$ |
| $1, s, 4, A, 5, A, 7$ | $B\$end$ | $r4$ |
| $1, s, 4, a, 6$ | $B\$end$ | $s8$ |
| $1, s, 4, a, 6, B, 8$ | $\$end$ | $r1$ |

4

**iii. (5 points)** Write down a production such that, if it were added to the grammar above, ocamlyacc would report a reduce/reduce conflict. Justify your answer by explaining what state would have a conflict and why.
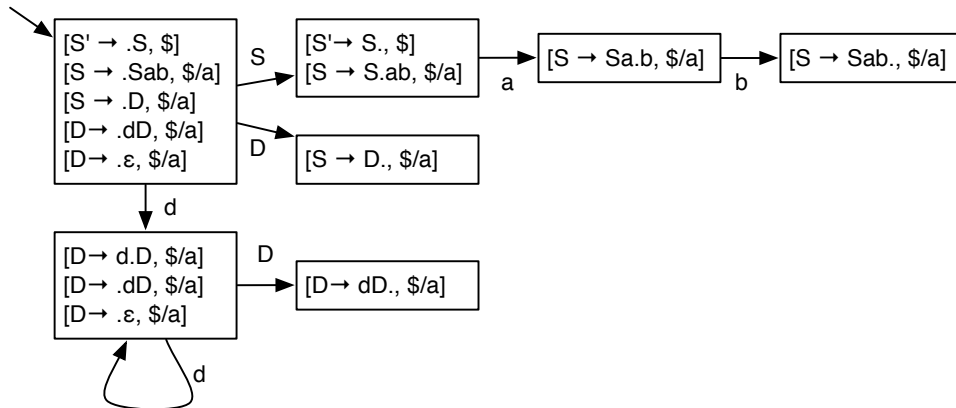
> **Answer:** There are many possible answers. For example, adding a duplicate production $s \rightarrow C$ creates a reduce/reduce conflict in state 3, because ocamlyacc can't decide between the two possible reductions for $C$.

**b. (15 points)** Draw the LR(1) parsing DFA for the following grammar:

$$
\begin{array}{rcl}
S & ::= & Sab \mid D \\
D & ::= & dD \mid \varepsilon
\end{array}
$$

**Answer:**

**Question 3. Operational semantics (15 points).**   Consider extending the lambda calculus to include *options*:

$$e \quad ::= \quad v \mid x \mid e\ e \mid \text{None} \mid \text{Some } e \mid \text{case } e \text{ of None} \rightarrow e, \text{Some } x \rightarrow e$$
$$v \quad ::= \quad n \mid \lambda x.e \mid \text{None} \mid \text{Some } v$$

Here are the operational semantics rules:

$$\frac{}{\text{BETA}}$$
$$\frac{}{(\lambda x.e_1)\ v_2 \rightarrow e_1[x \mapsto v_2]}$$

$$\text{LEFT} \quad \frac{e_1 \rightarrow e_1'}{e_1\ e_2 \rightarrow e_1'\ e_2}$$

$$\text{RIGHT} \quad \frac{e_2 \rightarrow e_2'}{v\ e_2 \rightarrow v\ e_2'}$$

$$\text{CASE} \quad \frac{e_0 \rightarrow e_0'}{(\text{case } e_0 \text{ of None} \rightarrow e_1, \text{Some } x \rightarrow e_2) \rightarrow (\text{case } e_0' \text{ of None} \rightarrow e_1, \text{Some } x \rightarrow e_2)}$$

$$\text{CASE-NONE} \quad \frac{}{(\text{case None of None} \rightarrow e_1, \text{Some } x \rightarrow e_2) \rightarrow e_1}$$

$$\text{CASE-SOME} \quad \frac{}{(\text{case Some } v \text{ of None} \rightarrow e_1, \text{Some } x \rightarrow e_2) \rightarrow e_2[x \mapsto v]}$$

**a. (5 points)** Following the above small-step rules, reduce the following term to a normal form. Show each step $e \rightarrow e'$ of reduction, but don't show the derivations underlying each step. For example, if we asked you to reduce $((\lambda x.x)\ (\lambda y.y))\ (\lambda z.z)$, your answer would be:

$$\begin{aligned} ((\lambda x.x)\ (\lambda y.y))\ (\lambda z.z) \quad &\rightarrow \\ (\lambda y.y)\ (\lambda z.z) \quad &\rightarrow \\ \lambda z.z \end{aligned}$$

Reduce this term:

$$((\lambda x.\text{case } x \text{ of None} \rightarrow (\lambda z.z), \text{Some } y \rightarrow (\lambda w.y))\ (\text{Some } 42))\ 5$$

   **Answer:**

$$\begin{aligned} ((\lambda x.\text{case } x \text{ of None} \rightarrow (\lambda z.z), \text{Some } y \rightarrow (\lambda w.y))\ (\text{Some } 42))\ 5 \quad &\rightarrow \\ (\text{case } (\text{Some } 42) \text{ of None} \rightarrow (\lambda z.z), \text{Some } y \rightarrow (\lambda w.y))\ 5 \quad &\rightarrow \\ (\lambda w.42)\ 5 \quad &\rightarrow \\ 42 \end{aligned}$$

**b. (5 points)** We left out one operational semantics rule above. Write down the missing rule, along with a program that cannot be evaluated without it.

   **Answer:** We left out the context rule for Some, and thus we can't evaluate a program like Some $(\lambda x.x)\ 5$. Here is the rule:

$$\text{SOME} \quad \frac{e \rightarrow e'}{\text{Some } e \rightarrow \text{Some } e'}$$

**c. (5 points)** Write down two *big-step* operational semantics rules for "case" that are equivalent to the small-step rules CASE, CASE-NONE, and CASE-SOME. As a reminder, here is a big-step rule for evaluating function application:

$$\frac{e_1 \to \lambda x.e \qquad e_2 \to v \qquad e[x \mapsto v] \to v'}{e_1 \ e_2 \to v'}$$

**Answer:**

$$\frac{e_0 \to \text{None} \qquad e_1 \to v}{(\text{case } e_0 \text{ of None} \to e_1, \text{Some } x \to e_2) \to v} \qquad \frac{e_0 \to \text{Some } v \qquad e_2[x \mapsto v] \to v'}{(\text{case } e_0 \text{ of None} \to e_1, \text{Some } x \to e_2) \to v'}$$

**Question 4. Type checking (25 points).** Now suppose we extend the simply typed lambda calculus to support options; thus we need to add *option types*:

$$e \quad ::= \quad v \mid x \mid e\,e \mid \mathsf{None} \mid \mathsf{Some}\,e \mid \mathsf{case}\,e\,\mathsf{of}\,\mathsf{None} \to e, \mathsf{Some}\,x{:}\,t \to e$$
$$v \quad ::= \quad n \mid \lambda x{:}t.e \mid \mathsf{None} \mid \mathsf{Some}\,v$$
$$t \quad ::= \quad int \mid t \to t \mid t\ option$$
$$A \quad ::= \quad \cdot \mid x{:}\,t, A$$

Here are some of the type rules for this language:

$$\mathrm{INT}\ \frac{}{A \vdash n : int} \qquad \mathrm{VAR}\ \frac{x \in dom(A)}{A \vdash x : A(x)} \qquad \mathrm{LAM}\ \frac{x{:}t, A \vdash e : t'}{A \vdash \lambda x{:}t.e : t \to t'}$$

$$\mathrm{APP}\ \frac{A \vdash e_1 : t \to t' \quad A \vdash e_2 : t}{A \vdash e_1\,e_2 : t'} \qquad \mathrm{SOME}\ \frac{A \vdash e : t}{A \vdash \mathsf{Some}\,e : t\ option}$$

**a. (10 points)** Fill in the types in the typing derivation below. You will need to determine what types $t_0$, $t_1$, and $t_2$ are. To save writing, write $i$ for *int* and *io* for *int option*.

$$t_0 = \boxed{int\ option \to int\ option} \qquad t_1 = \boxed{int} \qquad t_2 = \boxed{int\ option}$$

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{y{:}\boxed{t_1}, x{:}\boxed{t_0} \vdash x : \boxed{t_0} \quad\quad \cfrac{y{:}\boxed{t_1}, x{:}\boxed{t_0} \vdash y : \boxed{i}}{y{:}\boxed{t_1}, x{:}\boxed{t_0} \vdash \mathsf{Some}\,y : \boxed{io}}}{y{:}\boxed{t_1}, x{:}\boxed{t_0} \vdash x\,(\mathsf{Some}\,y) : \boxed{io}}}{x{:}\boxed{t_0} \vdash \lambda y{:}t_1.x\,(\mathsf{Some}\,y) : \boxed{(io \to io) \to i \to io}}}{\cdot \vdash (\lambda x{:}t_0.\lambda y{:}t_1.x\,(\mathsf{Some}\,y)) : \boxed{(io \to io) \to i \to io}}}{\cdot \vdash (\lambda x{:}t_0.\lambda y{:}t_1.x\,(\mathsf{Some}\,y))\,(\lambda z{:}t_2.z) : \boxed{i \to io}} \quad\quad \cfrac{\cfrac{z{:}\boxed{io} \vdash z : \boxed{io}}{\cdot \vdash (\lambda z{:}t_2.z) : \boxed{io \to io}}}{}}{\cdot \vdash (\lambda x{:}t_0.\lambda y{:}t_1.x\,(\mathsf{Some}\,y))\,(\lambda z{:}t_2.z)\,42 : \boxed{io}} \quad\quad \cfrac{}{\cdot \vdash 42 : \boxed{i}}}$$

**b. (5 points)** Write down the most general possible type rule for "None."

Answer:

$$\frac{}{A \vdash \text{None} : t \ option}$$

**c. (5 points)** Write down the most general possible type rule for "case."

Answer:

$$\frac{A \vdash e_1 : t_1 \qquad A \vdash e_2 : t' \qquad x{:}t, A \vdash e_3 : t'}{A \vdash \text{case } e_1 \text{ of None} \rightarrow e_2, \text{Some } x{:}t \rightarrow e_3 : t'}$$

**d. (5 points)** Using the following subset of the grammar that is missing lambda:

$$
\begin{array}{lcl}
e & ::= & v \mid x \mid e\ e \mid \text{None} \mid \text{Some } e \mid \text{case } e \text{ of None} \rightarrow e, \text{Some } x{:}t \rightarrow e \\
v & ::= & n \mid \text{None} \mid \text{Some } v \\
t & ::= & int \mid t \rightarrow t \mid t \ option \\
A & ::= & \cdot \mid x{:}t, A
\end{array}
$$

write down a term that is ill-typed according to the type rule from part c and yet does not get stuck at run time.

Answer: There are many possible answers, for example,

$$\text{case None of None} \rightarrow 3, \text{Some } x \rightarrow (3\ 4)$$

The type system will assume both branches are possible and report the error on the some branch, even though that code is not reachable at run time.

Another possible answer is

$$\text{case None of None} \rightarrow 3, \text{Some } 4 \rightarrow (\lambda x{:}int.x)$$

The type rule above forbids this program because the two branches' types don't match, yet the program above will not get stuck at run time.