CMSC451 Homework 3
Spring 2017 due March 2, 2017 at 3pm EST

For all algorithms, provide time complexity analysis as well as a formal proof of correctness. Homework solutions should be clearly written and electronically submitted via ELMS (`http://elms.umd.edu`) by the due date listed above.

1. Prove that in any undirected graph of n vertices ($n \geq 2$), there are always at least two vertices that have the same degree.

2. Suppose that an undirected graph $G = (V, E)$ contains two nodes s and t such that the distance between s and t is strictly greater than $n/2$. Show that there must exist some node v, not equal to either s or t, such that deleting v from G destroys all $s - t$ paths. Give an $O(m + n)$ algorithm to find such a node v.

3. A *cut vertex* of an undirected connected graph is one whose removal (and removal of its incident edges) disconnects the graph. Give an $O(m + n)$ algorithm to find all cut vertices of an undirected connected graph. (Hint: derive a relationship between cut vertices and low values.)

4. Given an unweighted tree $T = (V, E)$, let $\Gamma(T)$ denote the maximum distance between any two vertices in T. Give an $O(n)$ algorithm for computing $\Gamma(T)$.