Spring 2017 **CMSC 451: Homework 7** Chang & Khuller

May 4

(1) A *maximal matching* is a matching M such that no edges can be added to the matching, without destroying the property that it is a matching. In other words, for any edge e, $e \cup M$ is not a matching. First give a linear time algorithm for finding a maximal matching in a graph. Then prove that the size of the maximal matching is at least half the size of a maximum matching.

(2) Prove that a k-regular bipartite graph always has a perfect matching. (All nodes have degree exactly k.)

(3) Let $G = (X, Y, E)$ be a bipartite graph. Assume for simplicity that $|X| = |Y|$.

A subset $S \subseteq (X \cup Y)$ is called a *vertex cover* if for *each* edge $e = (x, y)$ in E either $x \in S$ or $y \in S$ (or both).

Let M^* be a maximum matching in G.

(a) Prove that if S is a vertex cover then $|S| \geq |M^*|$.

(b) Give a polynomial time algorithm to find the minimum cardinality vertex cover. Give a proof for its correctness.

(4)
- Suppose we know that problem X is NP-complete. Suppose we discover a polynomial time algorithm for X. Would that imply that the SATISFIABILITY problem can be solved in polynomial time? Explain your answer.
- Suppose we know that problem X belongs to NP. Suppose we discover a polynomial time algorithm for X. Would that imply that the SATISFIABILITY problem can be solved in polynomial time? Explain your answer.