May 11

Please fill out the Course Evaluation immediately. When you turn in your homework, sign a statement to the effect that you have done this. Your feedback is very important to both the University, and me personally.

(1) (a) Suppose we know that problem X is NP-complete. Suppose we discover a polynomial time algorithm for X. Would that imply that the SATISFIABILITY problem can be solved in polynomial time? Explain your answer.

(b) Suppose we know that problem X belongs to NP. Suppose we discover a polynomial time algorithm for X. Would that imply that the SATISFIABILITY problem can be solved in polynomial time? Explain your answer.

(2) The graph-3-coloring problem is defined as follows: Given a graph $G = (V, E)$ and three colors $\{1, 2, 3\}$, is there a way to assign a color to each node so that adjacent nodes always have different colors? Note that we can only use three colors.

Assume that Graph-3-coloring is NP-complete. Prove that graph-4-coloring is NP-complete.

(3) Give a polynomial reduction from Dominating Set to Set Cover. (Set cover is defined in Chapter 8 in the book.)

(4) HAMILTONIAN PATH PROBLEM: given a directed graph, does it contain a path that starts at some vertex and goes to some other vertex, going through each remaining vertex exactly once.

HAMILTONIAN CYCLE PROBLEM: given a directed simple graph, does it contain a directed simple cycle that goes through each vertex exactly once.

Assume that the HAMILTONIAN PATH PROBLEM is known to be NP-complete. Given this assumption, prove that the HAMILTONIAN CYCLE PROBLEM is NP-complete for directed graphs. (Show that the HAMILTONIAN CYCLE PROBLEM is in NP, and is also NP-hard.)

(5) Suppose there is an algorithm $Vertex - Cover(G; k)$ that runs in polynomial time and answers YES or NO, telling you correctly if graph G contains a vertex cover of size at most k or not.

Use this algorithm to actually find a minimum cardinality vertex cover. Your algorithm should run in polynomial time, and not only find the size of a smallest vertex cover, but also find the subset of vertices that form a vertex cover of minimum cardinality.