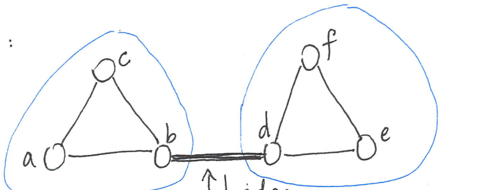
# 451 Lecture – Wed 2/13/2013 : 2-edge connectivity

Recall motivation for 2-edge connectivity (abbrev. 2-ec): how robust is a network? if one edge goes down, can overall graph connectivity still be maintained?

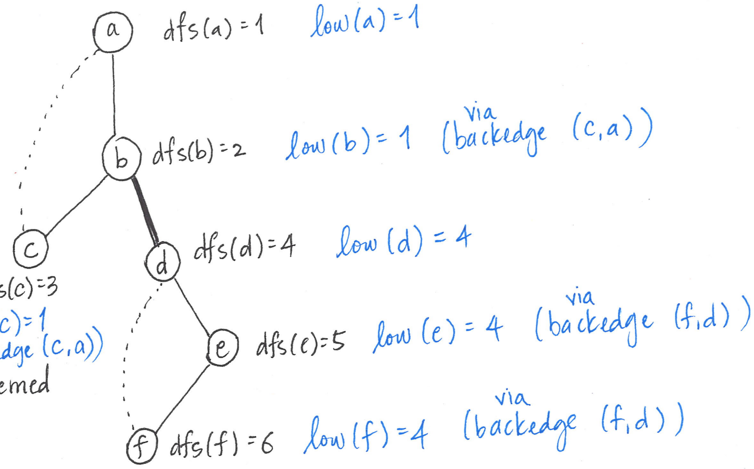
**Def'n:** for connected undirected graph $G = (V, E)$, edge $e$ is a **bridge** means $G \setminus \{e\}$ is disconnected.
↳ graph $G$ with edge $e$ deleted

**Def'n:** graph $G$ is **2-edge connected** means $G$ contains no bridges.

**Example:**




a DFS-tree of this graph:



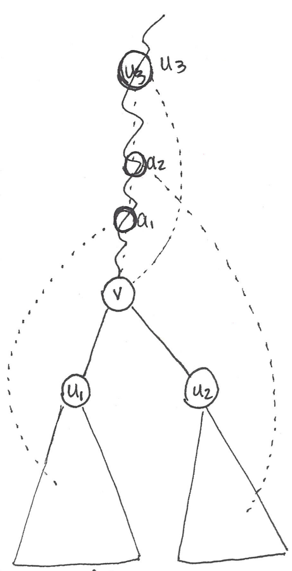

**Question:** how to identify bridges of a graph?

Consider DFS execution on a graph. (Let $dfs(v)$ denote discovery time of $v$. We'll not be concerned with finish times here.) Intuitively, bridges have property that there are no back edges from its descendants (or itself) to its ancestors.

**Dfn:** $low(v) = \min(\, dfs(v),\ \min\{dfs(x) : \exists \text{ backedge } (y,x) \text{ and } y \text{ is a descendant of } v\}\,)$

**Claim** (shown on Monday 2/11): bridges $= \{\text{edge } (\pi(u), u) : \pi(u) \neq NIL \text{ and } low(u) = dfs(u)\}$
i.e. $u$ is not root

Computing $low(\cdot)$ values requires a simple modification to DFS:




DFS(G)
- for all $v \in V$ do
  - color(v) ← white
  - π(v) ← NIL
  - dfs(v) ← ∞
  - low(v) ← ∞
- t ← 0
- for each $v \in V$ do
  - if color(v) is white do
    - DFS-Visit(v)
- for each $v \in V$ do
  - if $\pi(v) \neq NIL$ and $dfs(v) = low(v)$ do
    - output edge $(\pi(v), v)$ as a bridge

DFS-Visit(v)
- t ← t+1
- dfs(v) ← t
- color(v) ← gray
- low(v) ← dfs(v)
- for each $u \sim v$ do
  - if color(u) is white do  *(u is v's child in DFS tree)*
    - π(u) ← v
    - DFS-Visit(u)
    - low(v) ← min( low(v), low(u) )
  - else if $\pi(v) \neq u$ do  *(u is v's ancestor in DFS tree)*
    - low(v) ← min( low(v), dfs(u) )

example: low(v) starts as dfs(v). after we explore $u_1$, low(v) becomes $dfs(a_1)$. after we explore $u_2$, low(v) becomes $dfs(a_2)$. after we explore $u_3$ via a backedge, low(v) becomes $dfs(u_3)$.

∴ time to identify bridges is $O(m+n)$.