1. Evaluate $\prod_{k=1}^{n}\left(4 \cdot 3^k\right)$. Show your work.

2. Use a non-integral method to get upper and lower bounds for $\sum_{j=0}^{n} \sqrt{j}$ that are within a constant factor of each other. Show your work.

3. Use the integral method to get upper and lower bounds for $\sum_{j=0}^{n} \sqrt{j}$. The two values should have the same high level term. Show your work.

4. Assume you derive the following recurrence for the number of comparisons in Merge Sort for n a power of 2:
$$S(n) = 2S(n/2) + n - 1, \quad \text{and} \quad S(1) = 0$$
Assume you guess that, for constants a, b, c,
$$S(n) = an \lg n + bn + c$$
Use constructive induction to prove this and derive values for a, b, c. Show your work.

5. We showed in class (and in the book) that $\sum_{i=1}^{n} \frac{1}{i} \leq \lg n + O(1)$ using about $\lg n$ intervals. Show how to split each interval in half to get a better bound. Show your work.

6. Consider an array of size nine with the numbers $30, 50, 80, 40, 90, 20, 70, 10, 60$. Assume you execute quicksort using the version of partition from CLRS. Note that in this algorithm an element might exchange with itself (which counts as one exchange).
   (a) Show the array after the first partition. How many comparisons and exchanges are used?
   (b) Show the left side of the pivot after the next partition on the left side. How many comparisons are used? How many exchanges?
   (c) Show the right side of the original pivot after the next partition on the right side. How many comparisons are used? How many exchanges?