1. Suppose that we are trying to find the kth smallest number in a list of size n. Assume that we use the selection algorithm where we pivot on a random element. Make the simplifying assumption that at every recursive call, (the local) k is a random number between p and r (inclusive), so in the original recursive call k is a random number between 1 and n (inclusive). We will analyze the number of comparisons.
   (a) To warm up, assume that the pivot turns out to be always at the 1/4th mark, so in the original recursive call the pivot is at index $n/4$.
      i. There are actually three things that could happen in the original recursive call: we get lucky and $k = n/4$ so we are done, $k < n/4$, or $k > n/4$. What are these three probabilities? Just get the high order term for each probability.
      ii. Write a recurrence for the number of comparisons the algorithm uses (using the high order terms).
      iii. Solve the recurrence using constructive induction. Just get the high order term exactly.
   (b) Now assume that the pivot is random as above.
      i. There are actually three things that could happen in the original recursive call: we get lucky and $k = q$ so we are done, $k < q$, or $k > q$. For fixed q, what are these three probabilities?
      ii. Write a recurrence for the number of comparisons the algorithm uses. It should involve a summation.
      iii. Solve the recurrence using constructive induction. Just get the high order term exactly.
   (c) How do your answers from (a) and (b) compare?

2. Assume we use the selection algorithm from class (and from CLRS) but use columns of size 11 (rather than 5). Assume we use full *bubble sort* to find the median of each column.
   (a) Briefly list each step of the algorithm and how many comparisons the step takes.
   (b) Write a recurrence for the number of comparisons the algorithm uses.
   (c) Solve the recurrence using constructive induction. Just get the high order term exactly.
   (d) How does this value compare to what we got in class with columns of size 5?