CMSC 426, Computer Vision
Homework 0, Part 2: Theory
Due on: 11:59:59PM on Tuesday, February 6th

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Overview

The following questions give an overview of the mathematical concepts used in this course. These concepts are some of the foundation upon which the rest of Computer Vision is built– make sure you know them well!

Questions

1. A 1-D gaussian distribution is represented as

$$\mathcal{N}(\mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$$

   Now, consider that you have 2 gaussian distributions $\mathcal{N}(\mu_1, \sigma_1)$ and $\mathcal{N}(\mu_2, \sigma_2)$. Let $\mathcal{N}(\mu, \sigma) = \mathcal{N}(\mu_1, \sigma_1) \otimes \mathcal{N}(\mu_2, \sigma_2)$. Here $\otimes$ represents the convolution operation. Represent μ as a combination of μ_1, μ_2 and σ as a combination of σ_1, σ_2. What does this physically signify? **20 Pts**

2. Can three vectors in the xy plane have $u \cdot v < 0$, $v \cdot w < 0$ and $u \cdot w < 0$? **10 Pts**

3. Let $c \in R$. Suppose that A is an $n \times n$ matrix and that the sum of the entries in each column of A is c. Prove that c is an eigenvalue of A.
   Hint: Consider the sum of the row vectors of the matrix $A - cI$. **15 Pts**

4. If λ is an eigenvalue of A and X is the corresponding eigenvector, then prove that $\lambda - s$ is an eigenvalue of $A - sI$ for any scalar s and X is the corresponding eigenvector. **10 Pts**

5. For any two $n \times n$ matrices, say A and B:

(a) are real-symmetric matrices, both AB and BA always have the same eigenvalues. True or False?

(b) matrix B is invertible, AB and BA always have the same eigenvalues. True or False?

(c) matrix B is invertible, AB and BA always have the same eigenvectors. True or False?

Give support for all your answers. **20 Pts**

6. If rows of an $m \times n$ matrix A are linearly independent,

   (a) Is $Ax = b$ necessarily solvable?

   (b) If $Ax = b$ is solvable, is the solution necessarily unique?

   Explain. **15 Pts**

7. Show that if A is a non-singular matrix, and λ is an eigenvalue of A, then $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}. **10 Pts**

Submission Guidelines

Please submit a PDF of your answers **in the same zip file as your code and report from part 1**. Show all work and explain clearly. **Answers must be typeset in Latex, Word, LibreOffice, etc.– handwritten answers will not receive credit!**

Collaboration Policy

You are restricted to discuss the ideas with at most two other people. But the code you turn-in should be your own and if you **DO USE** (try not to and it is not permitted) other external codes/codes from other students - do cite them. For other honor codes refer to the CMSC426 Spring 2018 website.