School Choice: A Mechanism Design Approach

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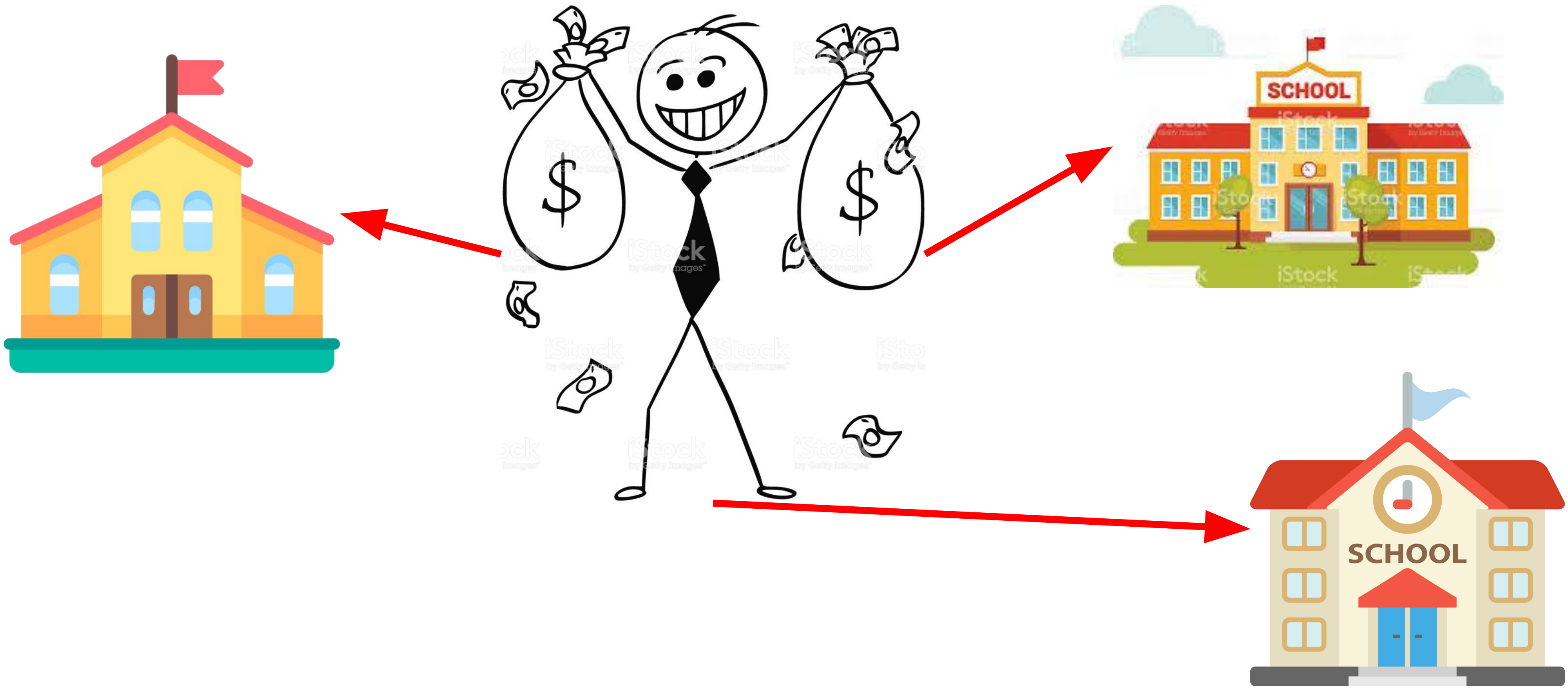
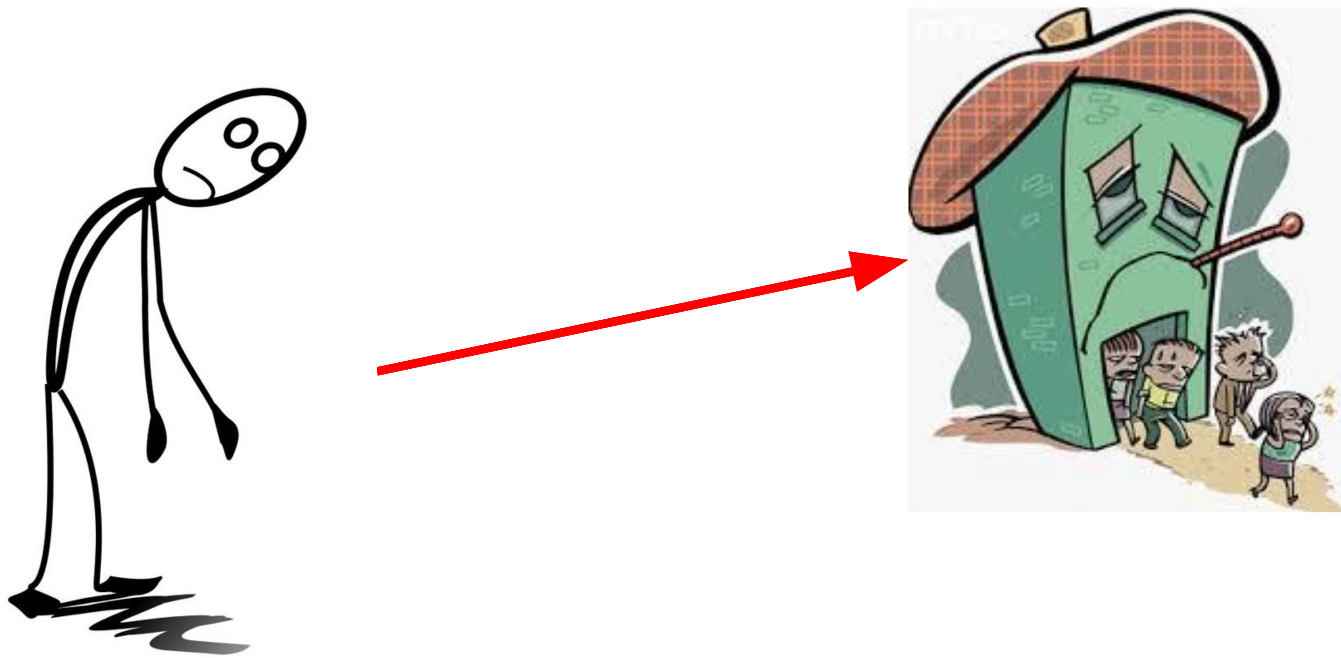
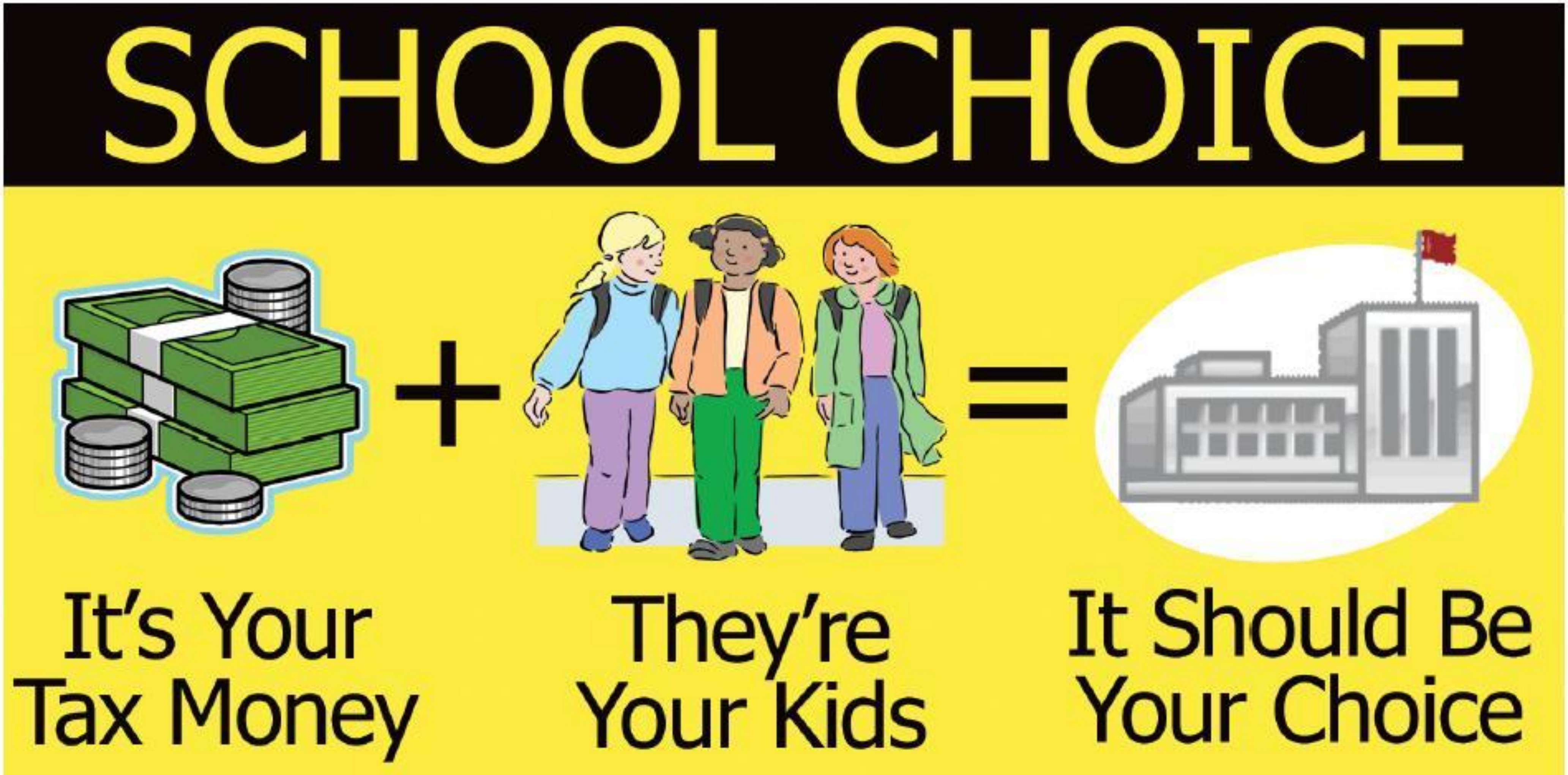
Before school choice: Wealthy



Before school choice: Not wealthy

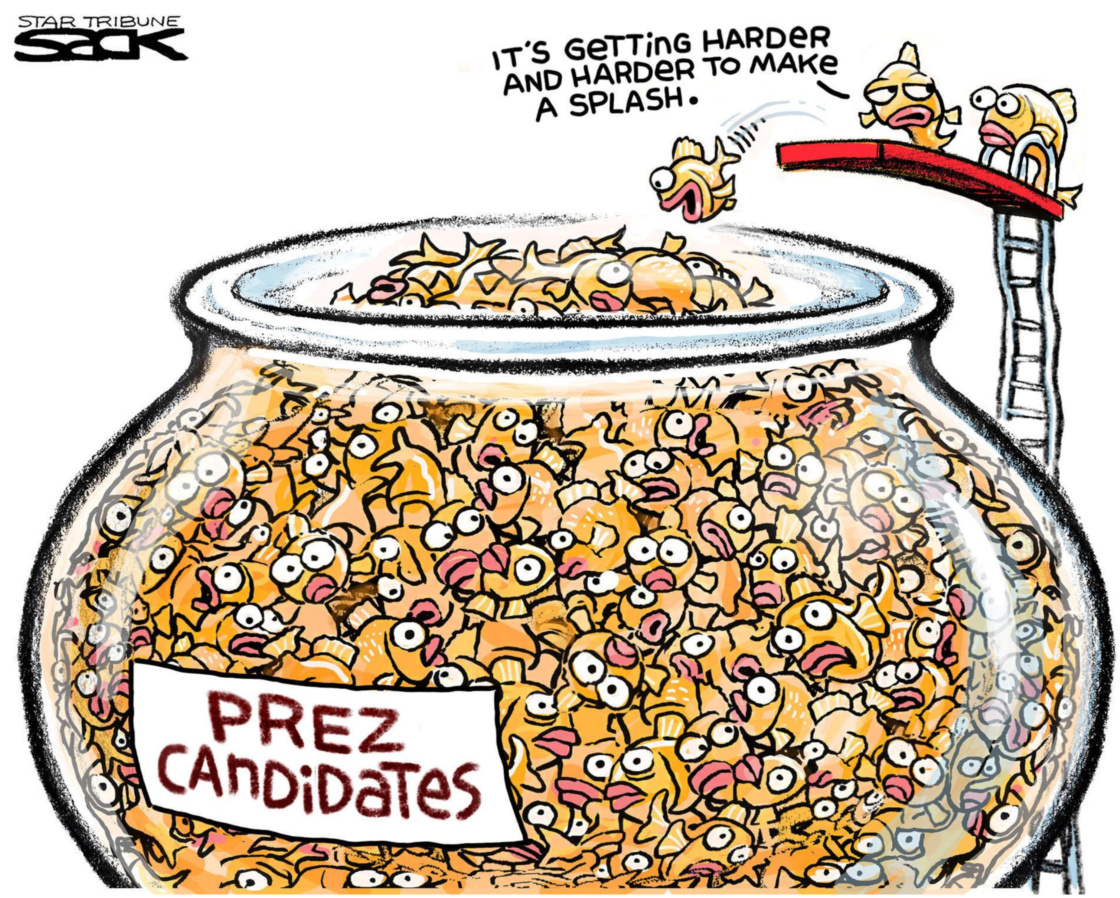


Why school choice?


SCHOOL CHOICE
It's Your Tax Money
+
They're Your Kids
=
It Should Be Your Choice

Why do we need an assignment mechanism?

It is not possible to assign each student to her top choice school, a central issue in school choice is the design of a *student assignment mechanism*



Problem setup:

Input:

- Number of students.
- Each student is to be assigned one seat at a school.
- Each school has a maximum capacity but there is no shortage in total number of seats.
- Each student has a strict priority preference.
- Each school has a strict priority ordering of all students.

Problem setup: School Priority ordering?!!!

Here, priorities do not represent school preferences but they are imposed by state or local laws, the priority ordering of a student can be different at different schools!

- Students who live in the attendance area of a school must be given priority for that school over students who do not live in the school’s attendance area.
- Siblings of students already attending a school must be given priority
- Students requiring a bilingual program must be given priority in schools that offer such programs.

Problem setup:

Output:

- Assignment of students to schools.
- Each student is assigned to exactly one school.
- No school is assigned to more students than its capacity.

Problem setup:

Goals:

An optimal mechanism should be:

- Pareto efficient
- Strategy-proof
- Envy free

Boston Student Assignment Mechanism

Boston Student Assignment Mechanism

1. Each student submits a preference ranking of the schools.
2. For each school a priority ordering is determined according to the following hierarchy:
   - First priority: sibling and walk zone.
   - Second priority: sibling.
   - Third priority: walk zone.
   - Fourth priority: other students.

3. Students in the same priority group are ordered based on a previously announced lottery.

Boston Student Assignment Mechanism

The final phase is the student assignment based on preferences and priorities:

- Round 1: Only the first choices of the students are considered.
- Round 2: Only the second choices of these students are considered.
- Round k: Only the k^{th} students are considered.

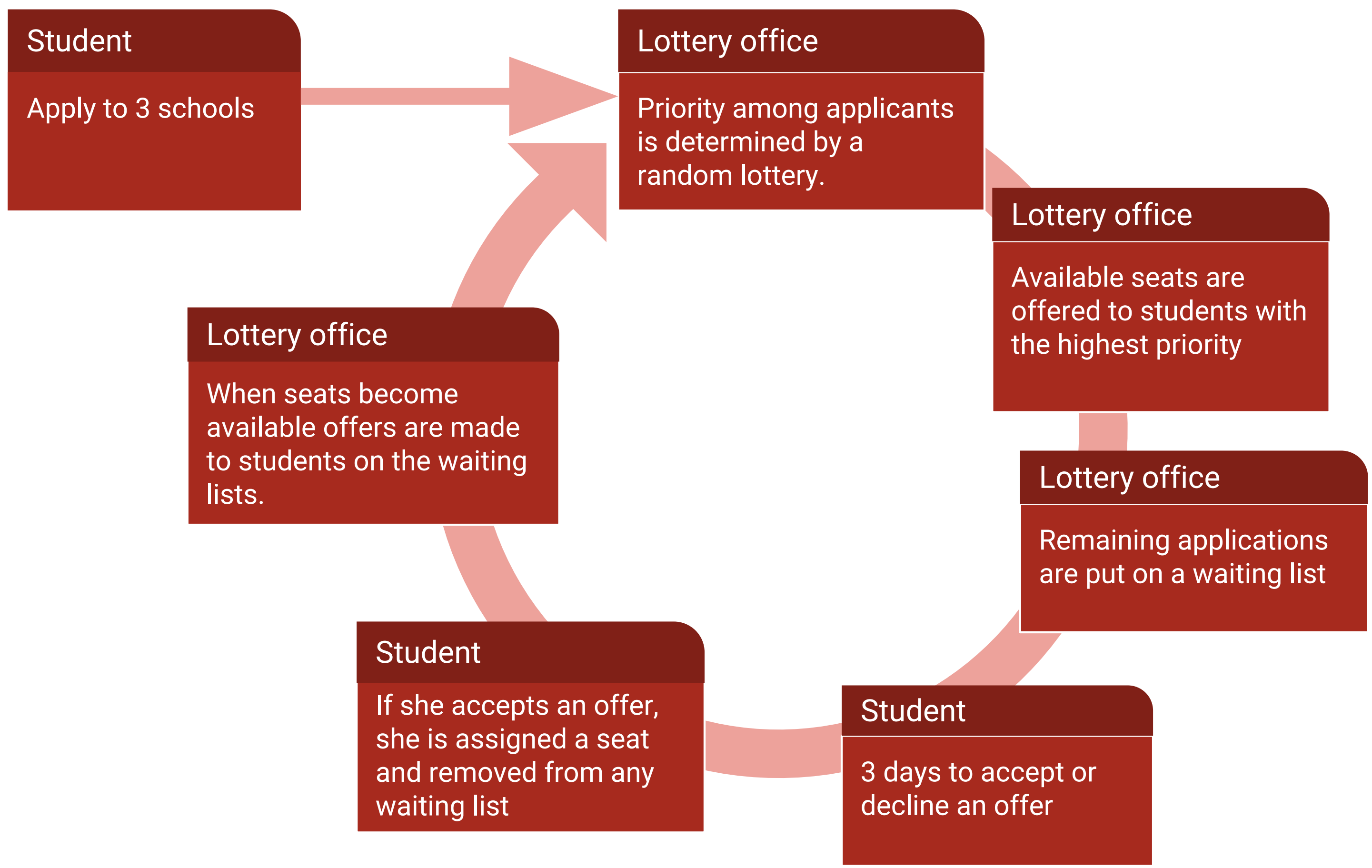
Boston Student Assignment Mechanism

- Boston student assignment mechanism is not strategy- proof.
- Even if a student has very high priority at school ***s***, unless she lists it as her top choice she loses her priority to students who have listed as their top choices.
- It may be optimal for some families to be strategic in listing their school choices. If a parent thinks that their favorite school is oversubscribed and they have a close second favorite, they may try to avoid “wasting” their first choice on a very popular school and instead list their number two school first.

Boston Student Assignment Mechanism

Columbus Student Assignment Mechanism

Columbus Student Assignment Mechanism



Columbus Student Assignment Mechanism

- The optimal application strategy of students is unclear under the Columbus student assignment mechanism.
- When a family gets an offer from its second or third choice, it is unclear whether the optimal strategy is declining this offer or accepting it.
- Another major difficulty with the Columbus student assignment mechanism concerns efficiency: Consider two students, each of whom hold an offer from the other’s first choice. Since they do not know whether they will receive better offers, they may as well accept these offers, and this in turn yields an inefficient matching.

Columbus Student Assignment Mechanism

Student assignment mechanisms: Dormitory rooms: Random serial dictatorship

Student assignment mechanisms: Dormitory rooms: Random serial dictatorship

- Order the students with a lottery and assign the first student her top choice, the next student her top choice among the remaining slots, and so on.
- This mechanism is not only *Pareto efficient,* but also *strategy-proof*

Student assignment mechanisms: Dormitory rooms: Random serial dictatorship

- A single lottery cannot be used to allocate school seats to students.
- It is this school-specific priority feature of the problem that complicates the student assignment process.
- A student assignment mechanism should be flexible enough to give students different priorities at different schools.

Student assignment mechanisms: Dormitory rooms: Random serial dictatorship

Student assignment mechanisms: College Admissions

Student assignment mechanisms: College Admissions

- The central difference between the college admissions and school choice is that in college admissions, schools themselves are agents which have preferences over students.
- Whereas in school choice, schools are merely “objects” to be consumed by the students.
- A student should not be rejected by a school because of her personality or ability level

Student assignment mechanisms: College Admissions

- **College Admissions:** Student-college pair (***i, s***) where student ***i*** prefers college ***s*** to her assignment and college ***s*** prefers student ***i*** to one or more of its admitted students.
- **School choice:** Student-school pair (***i, s***) where student ***i*** prefers school ***s*** to her assignment and she has higher priority than some other student who is assigned a seat at school **s**.

Student assignment mechanisms: College Admissions

- A stable matching in the context of college admissions eliminates justified envy in the context of school choice.

- **Good news:** There exists a stable matching which is preferred to any stable matching by every student in the context of college admissions **Gale and Shapley**

Gale-Shapley Student Optimal Stable Mechanism

Step 1:

- Each student proposes to her first choice.
- Each school tentatively assigns its seats to its proposers one at a time following their priority order.
- Any remaining proposers are rejected.

Gale-Shapley Student Optimal Stable Mechanism

In general Step k:

  - Each student who was rejected in the previous step proposes to her next choice.
  - Each school considers the students it has been holding together with its new proposers and tentatively assigns its seats to these students one at a time following their priority order.
  - Any remaining proposers are rejected.
- The algorithm terminates when no student proposal is rejected and each student is assigned her final tentative assignment.

Gale-Shapley Student Optimal Stable Mechanism

Good news:

- It is strategy-proof.
- Pareto-dominates any other mechanism that eliminates justified envy.

Gale-Shapley Student Optimal Stable Mechanism

Good news:

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Bad news:

- There is a potential trade-off between stability and Pareto efficiency.

Gale-Shapley Student Optimal Stable Mechanism

Example:

- There are three students i_1, i_2, i_3, and three schools s_1,s_2,s_3, each of which has only one seat.
- The priorities of schools and the preferences of students are as follows:

$$
\begin{array}{llll}
s_1 : i_1 - i_3 - i_2 & \qquad & i_1 : s_2 \; s_1 \; s_3 \\
s_2 : i_2 - i_1 - i_3 & \qquad & i_2 : s_1 \; s_2 \; s_3 \\
s_3 : i_2 - i_1 - i_3 & \qquad & i_3 : s_1 \; s_2 \; s_3 .
\end{array}
$$

Gale-Shapley Student Optimal Stable Mechanism

Example:

$$\begin{array}{llll} s_1 : i_1 - i_3 - i_2 & & i_1 : s_2 \;\; s_1 \;\; s_3 \\ s_2 : i_2 - i_1 - i_3 & & i_2 : s_1 \;\; s_2 \;\; s_3 \\ s_3 : i_2 - i_1 - i_3 & & i_3 : s_1 \;\; s_2 \;\; s_3 . \end{array}$$

i_1	s_2
i_2	
i_3	

Gale-Shapley Student Optimal Stable Mechanism

Example:

$$
\begin{array}{llll}
s_1 : i_1 - i_3 - i_2 & & i_1 : s_2 \; s_1 \; s_3 \\
s_2 : i_2 - i_1 - i_3 & & i_2 : s_1 \; s_2 \; s_3 \\
s_3 : i_2 - i_1 - i_3 & & i_3 : s_1 \; s_2 \; s_3 .
\end{array}
$$

i_1	s_2
i_2	s_1
i_3	

Gale-Shapley Student Optimal Stable Mechanism

Example:

$$
\begin{array}{llll}
s_1 : i_1 - i_3 - i_2 & \quad i_1 : s_2 \; s_1 \; s_3 \\
s_2 : i_2 - i_1 - i_3 & \quad i_2 : s_1 \; s_2 \; s_3 \\
s_3 : i_2 - i_1 - i_3 & \quad i_3 : s_1 \; s_2 \; s_3 .
\end{array}
$$

i_1	s_2
i_2	s_1
i_3	s_1

Gale-Shapley Student Optimal Stable Mechanism

Example:

$$\begin{array}{llll} s_1 : i_1 - i_3 - i_2 & & i_1 : s_2 \ s_1 \ s_3 \\ s_2 : i_2 - i_1 - i_3 & & i_2 : s_1 \ s_2 \ s_3 \\ s_3 : i_2 - i_1 - i_3 & & i_3 : s_1 \ s_2 \ s_3 . \end{array}$$

i_1	$\cancel{s_2}$	
i_2	$\cancel{s_1}$	s_2
i_3	s_1	

Gale-Shapley Student Optimal Stable Mechanism

Example:

$$
\begin{array}{llll}
s_1 : i_1 - i_3 - i_2 & \quad i_1 : s_2 \; s_1 \; s_3 \\
s_2 : i_2 - i_1 - i_3 & \quad i_2 : s_1 \; s_2 \; s_3 \\
s_3 : i_2 - i_1 - i_3 & \quad i_3 : s_1 \; s_2 \; s_3 .
\end{array}
$$

i_1	$\cancel{s_2}$	s_1
i_2	$\cancel{s_1}$	s_2
i_3	$\cancel{s_1}$	

Gale-Shapley Student Optimal Stable Mechanism

Example:

$$
\begin{array}{llll}
s_1 : i_1 - i_3 - i_2 & \quad & i_1 : s_2 \;\; s_1 \;\; s_3 \\
s_2 : i_2 - i_1 - i_3 & \quad & i_2 : s_1 \;\; s_2 \;\; s_3 \\
s_3 : i_2 - i_1 - i_3 & \quad & i_3 : s_1 \;\; s_2 \;\; s_3 .
\end{array}
$$

i_1	~~s_2~~	s_1
i_2	~~s_1~~	s_2
i_3	~~s_1~~	s_3

Gale-Shapley Student Optimal Stable Mechanism

Example:

$$
\begin{array}{llll}
s_1 : i_1 - i_3 - i_2 & \quad & i_1 : s_2 \;\; s_1 \;\; s_3 \\
s_2 : i_2 - i_1 - i_3 & \quad & i_2 : s_1 \;\; s_2 \;\; s_3 \\
s_3 : i_2 - i_1 - i_3 & \quad & i_3 : s_1 \;\; s_2 \;\; s_3 .
\end{array}
$$

i_1	s_1
i_2	s_2
i_3	s_3

But this matching is Pareto-dominated by

i_1	s_2
i_2	s_1
i_3	s_3

Top trading cycles mechanism

Top trading cycles mechanism

- A competing mechanism which is Pareto efficient but which does not completely eliminate justified envy.
- Suppose that if student i_1 has higher priority than student i_2 for school ***s,*** that does not necessarily mean that she is entitled a seat at school **s** before student i_2. It rather represents the opportunity to get into school ***s***. If i_1 has higher priority than i_2, then she has a better opportunity to get into school ***s***, other things being equal.

Top trading cycles mechanism

Step 1:

- Assign a counter for each school which keeps track of how many seats are still available at the school.
- Each student points to her favorite school under her announced preferences.
- Each school points to the student who has the highest priority for the school.
- There is at least one cycle.
- Every student in a cycle is assigned a seat at the school she points to and is removed.
- The counter of each school in a cycle is reduced by one and if it reduces to zero, the school is also removed.

Top trading cycles mechanism

In general Step k:

- Each remaining student points to her favorite school among the remaining schools.
- Each remaining school points to the student with highest priority among the remaining students.
- There is at least one cycle.
- Every student in a cycle is assigned a seat at the school that she points to and is removed.
- The counter of each school in a cycle is reduced by one and if it reduces to zero the school is also removed.

Top trading cycles mechanism

Example:

- There are eight students $i_1, \dots, i_8$ and four schools $s_1, \dots, s_4$. Schools s_1, s_2 have two seats each and schools s_3, s_4 have three seats each.
- The priorities of the schools and the preferences of the students are as follows:

$$
\begin{aligned}
s_1 &: i_1 - i_2 - i_3 - i_4 - i_5 - i_6 - i_7 - i_8 \\
s_2 &: i_3 - i_5 - i_4 - i_8 - i_7 - i_2 - i_1 - i_6 \\
s_3 &: i_5 - i_3 - i_1 - i_7 - i_2 - i_8 - i_6 - i_4 \\
s_4 &: i_6 - i_8 - i_7 - i_4 - i_2 - i_3 - i_5 - i_1
\end{aligned}
$$

i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
s_2	s_1	s_3	s_3	s_1	s_4	s_1	s_1
s_1	s_2	s_2	s_4	s_3	s_1	s_2	s_2
s_3	s_3	s_1	s_1	s_4	s_2	s_3	s_4
s_4	s_4	s_4	s_2	s_2	s_3	s_4	s_3

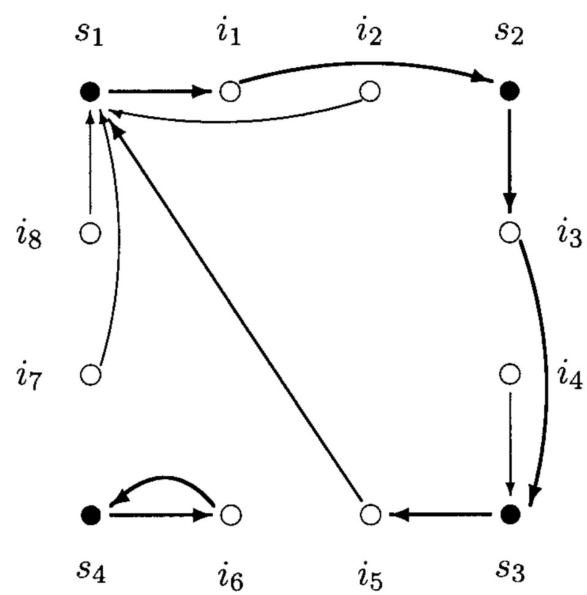
Top trading cycles mechanism

Step 1:

$s_1=2$, $s_2=2$, $s_3=3$, $s_4=3$

$s_1 : i_1 - i_2 - i_3 - i_4 - i_5 - i_6 - i_7 - i_8$
$s_2 : i_3 - i_5 - i_4 - i_8 - i_7 - i_2 - i_1 - i_6$
$s_3 : i_5 - i_3 - i_1 - i_7 - i_2 - i_8 - i_6 - i_4$
$s_4 : i_6 - i_8 - i_7 - i_4 - i_2 - i_3 - i_5 - i_1$

i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
s_2	s_1	s_3	s_3	s_1	s_4	s_1	s_1
s_1	s_2	s_2	s_4	s_3	s_1	s_2	s_2
s_3	s_3	s_1	s_1	s_4	s_2	s_3	s_4
s_4	s_4	s_4	s_2	s_2	s_3	s_4	s_3



Output:

$i_1 \Rightarrow s_2$

$i_3 \Rightarrow s_3$

$i_5 \Rightarrow s_1$

$i_6 \Rightarrow s_4$

$s_1=1$, $s_2=1$, $s_3=2$, $s_4=2$

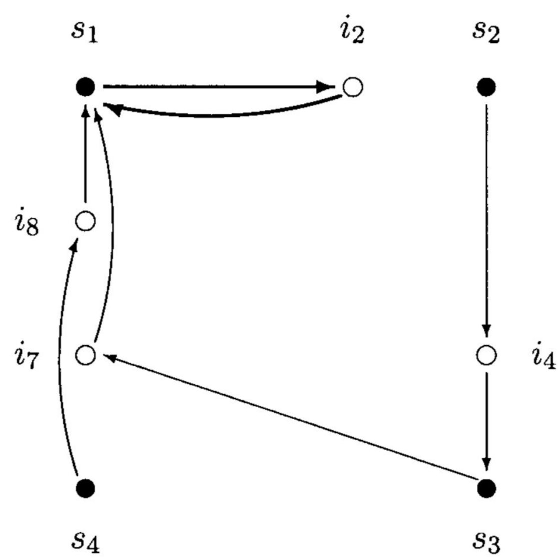
Top trading cycles mechanism

Step 2:

$s_1=1$, $s_2=1$, $s_3=2$, $s_4=2$

$s_1 : \cancel{i_1} - i_2 - \cancel{i_3} - i_4 - \cancel{i_5} - \cancel{i_6} - i_7 - i_8$
$s_2 : \cancel{i_3} - \cancel{i_5} - i_4 - i_8 - i_7 - i_2 - \cancel{i_1} - \cancel{i_6}$
$s_3 : \cancel{i_5} - \cancel{i_3} - \cancel{i_1} - i_7 - i_2 - i_8 - \cancel{i_6} - i_4$
$s_4 : \cancel{i_6} - i_8 - i_7 - i_4 - i_2 - \cancel{i_3} - \cancel{i_5} - \cancel{i_1}$

~~i_1~~	i_2	~~i_3~~	i_4	~~i_5~~	~~i_6~~	i_7	i_8
~~s_2~~	s_1	~~s_3~~	s_3	~~s_1~~	~~s_4~~	s_1	s_1
~~s_1~~	s_2	~~s_2~~	s_4	~~s_3~~	~~s_1~~	s_2	s_2
~~s_3~~	s_3	~~s_1~~	s_1	~~s_4~~	~~s_2~~	s_3	s_4
~~s_4~~	s_4	~~s_4~~	s_2	~~s_2~~	~~s_3~~	s_4	s_3



Output:

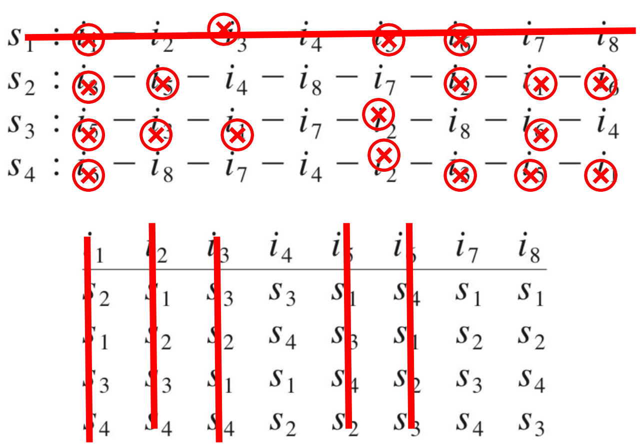
$i_2 \Rightarrow s_1$

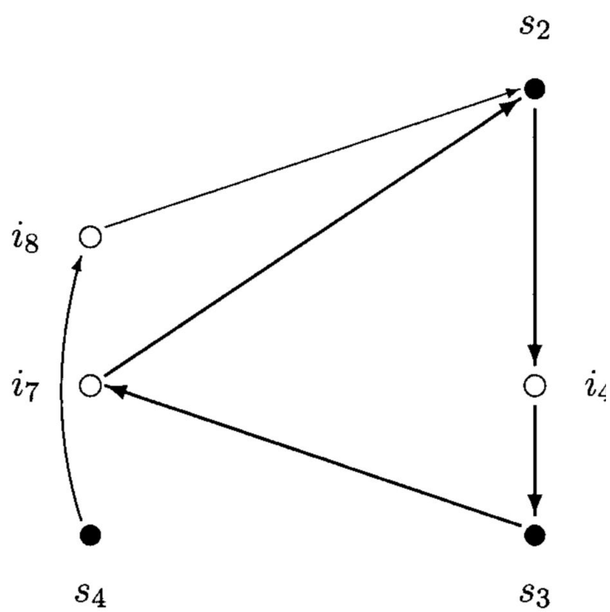
$s_1=0$, $s_2=1$, $s_3=2$, $s_4=2$

Top trading cycles mechanism

Step 3:

$s_1=0$, $s_2=1$, $s_3=2$, $s_4=2$





Output:

$i_4 \Rightarrow s_3$

$i_7 \Rightarrow s_2$

$s_1=0$, $s_2=0$, $s_3=1$, $s_4=2$

Top trading cycles mechanism

Step 4:

$s_1=0$, $s_2=0$, $s_3=1$, $s_4=2$

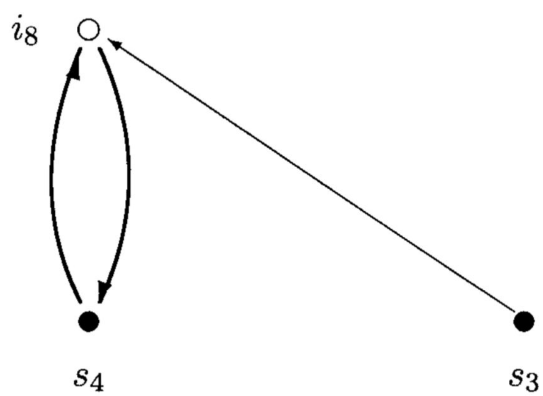
$s_1 : i_1 - i_2 - i_3 \quad i_4 \quad i_5 \quad i_6 \quad i_7 \quad i_8$

$s_2 : i_{[illegible]} - i_{[illegible]} - i_4 - i_8 - i_7 - i_2 - i_1 - i_{[illegible]}$

$s_3 : i_5 - i_3 - i_1 - i_7 - i_2 - i_8 - i_6 - i_4$

$s_4 : i_6 - i_8 - i_7 - i_4 - i_2 - i_3 - i_5 - i_1$

i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
s_2	s_1	s_3	s_3	s_1	s_4	s_1	s_1
s_1	s_2	s_2	s_4	s_3	s_1	s_2	s_2
s_3	s_3	s_1	s_1	s_4	s_2	s_3	s_4
s_4	s_4	s_4	s_2	s_2	s_3	s_4	s_3



Output:

$i_8 \Rightarrow s_4$

$s_1=0$, $s_2=0$, $s_3=1$, $s_4=1$

Top trading cycles mechanism

Good news:

- It is strategy-proof.
- Pareto-dominates any other mechanism that eliminates justified envy.

Which Mechanism Shall Be Chosen?

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- Both mechanisms are strategy-proof, so the choice between them depends on the structure and interpretation of the priorities.

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Which Mechanism Shall Be Chosen?

- Both mechanisms are strategy-proof, so the choice between them depends on the structure and interpretation of the priorities.

It depends on the application

- In some applications, policy makers may rank complete elimination of justified envy before full efficiency, and Gale-Shapley student optimal stable mechanism can be used in those cases.
- In other applications, the top trading cycles mechanism may be more appealing.

Controlled choice mechanism

Controlled choice mechanism

- One of the major concerns about the implementation of school choice plans is that they may result in racial and ethnic segregation at schools.
- Because of these concerns, choice plans in some districts are limited by court ordered desegregation guidelines.
- Both Gale-Shapley student optimal stable mechanism and the top trading cycles mechanism can be easily modified to accommodate controlled choice constraints by imposing type-specific quotas.

Controlled choice mechanism

- Suppose that there are different types of students and each student belongs to one type.
- If the controlled choice constraints are perfectly rigid then there is no need to modify the mechanisms.
- For each type of students, one can separately implement the mechanism in order to allocate the seats that are reserved exclusively for that type.
- When the controlled choice constraints are flexible, modification are need to both Gale-Shapley student optimal stable and top trading cycles mechanisms.

Gale-Shapley Student Optimal Stable Mechanism with Type-Specific Quotas

Step K:

- Each student who was rejected in the previous step proposes to her next choice.
- Each school considers the students it has been holding together with its new proposers and tentatively assigns its seats to these students one at a time following their priority order.
- ***If the quota of a type fills, the remaining proposers of that type are rejected and the tentative assignment proceeds with the students of the other types.***
- Any remaining proposers are rejected.

Gale-Shapley Student Optimal Stable Mechanism with Type-Specific Quotas

This modified mechanism satisfies the following version of the fairness requirement:

If there is an unmatched student-school pair ***(i, s)*** where student ***i*** prefers school ***s*** to her assignment and she has higher priority than some other student ***j*** who is assigned a seat at school ***s***, then:

1. Students ***i*** and ***j*** are of different types
2. The quota for the type of student ***i*** is full at school ***s***.

Top Trading Cycles Mechanism with Type-Specific Quotas

Step K:

- Each remaining student points to her favorite remaining school among those which have room for her type.
- Each remaining school points to the student with the highest priority among remaining students.
- There is at least one cycle.
- Every student in a cycle is assigned a seat at the school that she points to and is removed.
- The counter of each school in a cycle is reduced by one and depending on the student it is assigned to.
- ***The associated type-specific counter is reduced by one as well.***
- All other counters stay put.
- In case the counter of a school reduces to zero, the school is removed.
- If there is at least one remaining student, then we proceed with the next step.

Questions?