$\neg \forall x : P(x) \qquad \exists x \neg P(x)$

$\neg(x \vee y)$

$\exists x : \neg(x \vee y)$

$\equiv \exists x : \neg x \wedge \neg y$

$\neg \exists x : P(x) \equiv \forall x : \neg P(x)$

$\forall x \; \exists y (xy = 1)$

Negation

$\neg (\forall x \; \exists y (xy = 1))$

$\equiv \exists x \neg \exists y (xy = 1)$

$\equiv \exists x \forall y \neg (xy = 1)$

$\equiv \exists x \forall y (xy \neq 1)$

$$\exists m \forall a \exists f \left(P(m,f) \wedge Q(f,a) \right)$$

Negation

$$\equiv \neg \exists m \forall a \exists f \left(P(m,f) \wedge Q(f,a) \right)$$

$$\equiv \forall m \neg \forall a \exists f \left(P(m,f) \wedge Q(f,a) \right)$$

$$\equiv \forall m \exists a \neg \exists f \left(P(m,f) \wedge Q(f,a) \right)$$

$$\equiv \forall m \exists a \forall f \neg \left(P(m,f) \wedge Q(f,a) \right)$$

$$\equiv \forall m \exists a \forall f \left(\neg P(m,f) \vee \neg Q(f,a) \right)$$

$$x \longrightarrow y$$

$$\neg \left(\neg x \vee y \right)$$