

CMSC 250

Discrete Structures

Review

Simple Translation

Definition

p: $0 < x$

q: $x < 5$

r: $x = 5$

Example

- $x \leq 5$

Simple Translation

Definition

p: $0 < x$

q: $x < 5$

r: $x = 5$

Example

- $x \leq 5$

$q \vee r$

Simple Translation

Definition

p: $0 < x$

q: $x < 5$

r: $x = 5$

Example

- $x \leq 5$

$$q \vee r$$

- $0 < x < 5$

Simple Translation

Definition

p: $0 < x$

q: $x < 5$

r: $x = 5$

Example

- $x \leq 5$

$$q \vee r$$

- $0 < x < 5$

$$p \wedge q$$

- $0 < x \leq 5$

Simple Translation

Definition

$p: 0 < x$

$q: x < 5$

$r: x = 5$

Example

- $x \leq 5$

$$q \vee r$$

- $0 < x < 5$

$$p \wedge q$$

- $0 < x \leq 5$

$$p \wedge (q \vee r)$$

Common Logical Equivalencies

Example

$$\neg\neg p \equiv p$$

Double negation

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

De Morgan's law

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

De Morgan's law

$$p \wedge q \equiv q \wedge p$$

Commutativity of AND

$$p \vee q \equiv q \vee p$$

Commutativity of OR

$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$$

Associativity of AND

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

Associativity of OR

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

AND distributes over OR

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

OR distributes over AND

$$p \rightarrow q \equiv \neg p \vee q$$

Equivalence of implication and OR

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Contraposition

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

Expansion of if and only if

$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$

Inverse of if and only if

$$p \leftrightarrow q \equiv q \leftrightarrow p$$

Commutativity of if and only if

Truth Table

- $p \rightarrow q$
- $p \leftrightarrow q$
- Is $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$?

Contrapositive

Example

If I am a human then I am a mammal

Contrapositive

Example

If I am a human then I am a mammal

If I am not a mammal then I am not a human.

Definition

$p \rightarrow q$

Contrapositive: $\neg q \rightarrow \neg p$

Inverse

Example

If I am a human then I am a mammal

Inverse

Example

If I am a human then I am a mammal

If I am not human then I am not a mammal.

Definition

$p \rightarrow q$

Inverse: $\neg p \rightarrow \neg q$

Converse

Example

If I am human then I am a mammal

Converse

Example

If I am human then I am a mammal

If I am a mammal then I am human.

Definition

$p \rightarrow q$

Inverse: $q \rightarrow p$

Negations

Example

The units digit of 4^{67} is 4 or it is 6.

Negations

Example

The units digit of 4^{67} is 4 or it is 6.

The dollar is at an all-time high and the stock market is at a record low.

True or false

Example

Is the follow statement true or false?

If $0 = 1$ then $1 = 2$

Logical Equivalence

Example

Show that the following propositions are logically equivalent:

- $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$
- $(q \vee r) \rightarrow p \equiv (q \rightarrow p) \wedge (r \rightarrow p)$

Boolean circuits

Example

<i>P</i>	<i>Q</i>	<i>R</i>	<i>S</i>
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

Translations

Example

- If somebody talks, it will be in the news tomorrow

Translations

Example

- If somebody talks, it will be in the news tomorrow
 $\exists x P(x) \rightarrow Q$

Helpful Examples

$L(x,y)$: x likes y

Example

Everything likes everything

Helpful Examples

$L(x,y)$: x likes y

Example

Everything likes everything

$$\forall x \forall y L(x, y)$$

Helpful Examples

$L(x,y)$: x likes y

Example

Everything likes everything

$$\forall x \forall y L(x, y)$$

Everything is liked by everything

Helpful Examples

$L(x,y)$: x likes y

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Everything is liked by everything

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$L(x,y)$: x likes y

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Everything likes everything

$$\forall x \forall y L(x, y)$$

Everything is liked by everything

$$\forall y \forall x L(x, y)$$

Something likes something

Helpful Examples

$L(x,y)$: x likes y

Example

Everything likes everything

$$\forall x \forall y L(x, y)$$

Everything is liked by everything

$$\forall y \forall x L(x, y)$$

Something likes something

$$\exists x \exists y L(x, y)$$

Helpful Examples

$L(x,y)$: x likes y

Example

Everything likes everything

$$\forall x \forall y L(x, y)$$

Everything is liked by everything

$$\forall y \forall x L(x, y)$$

Something likes something

$$\exists x \exists y L(x, y)$$

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Everything is liked by everything

$$\forall y \forall x L(x, y)$$

Something likes something

$$\exists x \exists y L(x, y)$$

Something is liked by something

$$\exists y \exists x L(x, y)$$

Nothing likes anything

$$\forall x \forall y \neg L(x, y)$$

Helpful Examples

$L(x,y)$: x likes y

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Helpful Examples

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