CMSC 250
Discrete Structures

Sequences, Summations and Products

Practice finding an explicit formula

Figure out the formula for this sequence:

$$1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \ldots$$

Summation & product

- Sum of specified items
  $\sum_{k=1}^{6} 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$
- Product of specified items
  $\prod_{k=1}^{5} 2k = 2(1) \times 2(2) \times 2(3) \times 2(4) \times 2(5)$

Variable ending point

- n as the index of the final term

$$\sum_{k=0}^{n} \frac{k+1}{n+k}$$

Evaluate for

- $n = 2$
- $n = 3$

Nesting of sum/product

- Variations (same or different ?)

$$\sum_{j=1}^{n}\sum_{i=1}^{m_j} Y_{ij}^2 \qquad \sum_{j=1}^{n}\left(\sum_{i=1}^{m_j} Y_{ij}\right)^2 \qquad \left(\sum_{j=1}^{n}\sum_{i=1}^{m_j} Y_{ij}\right)^2$$

Telescoping series

$$\sum_{k=1}^{n} \left(\frac{k}{k+1} - \frac{k+1}{k+2} \right)$$

$$\prod_{i=1}^{n} \left(\frac{i}{i+1} \right)$$

Merging and Splitting

- Summations

$$\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k)$$

$$\sum_{k=m}^{n} a_k = \sum_{k=m}^{i} a_k + \sum_{k=i+1}^{n} a_k$$

- Products

$$\prod_{k=m}^{n} a_k \times \prod_{k=m}^{n} b_k = \prod_{k=m}^{n} (a_k \times b_k)$$

$$\prod_{k=m}^{n} a_k = \prod_{k=m}^{i} a_k \times \prod_{k=i+1}^{n} a_k$$

Distribution

$$c \times \sum_{k=m}^{n} a_k = \sum_{k=m}^{n} (c \times a_k)$$

Change of variable

$$\sum_{k=0}^{6} \frac{1}{k+1}$$

Factorial

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$$
$$0! = 1$$
$$n! = n \times (n-1)!$$