CMSC 250
Discrete Structures

Constructive Induction

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Example

For all $n \geq 1$

$$\sum_{i=1}^{n} 4i - 2$$

Constructive induction

$$a_0 = 2 \qquad a_1 = 7$$

$$(\forall k \in Z^{\geq 2})\left[a_k = 12a_{k-1} + 3a_{k-2}\right]$$

We suspect that this recurrence is bounded by some exponential function of the form AB^n, where A and B are integers:

$$(\forall n \in Z^{\geq 0})\left[a_n \leq A \cdot B^n\right]$$

We would like to find the *smallest* integers A and B that make this work.

Introduction to Set Theory

Set Definitions

Definition

A *set* is an unordered collection of elements.
name of set = {list of elements, or a description of the elements}

Example

S = {a,b,c,d}, $a \in A$ and $e \notin A$
A = {1,2,3}
B = $\{x \in Z | -4 < x < 4\}$
C = $\{x \in Z^+ | -4 < x < 4\}$

More Set Concepts

Definition

- The universal set, U, is the set consisting of all possible elements in some particular situation under consideration.
- A set can be finite or can be infinite.
- For a set S, n(S) or $|S|$ are used to refer to the cardinality of S, which is the number of elements in S.

Subset

Definition

- $A \subseteq B \leftrightarrow (\forall x \in U)[x \in A \rightarrow x \in B]$
  A is contained in B
  B contains A
- $A \not\subseteq B \leftrightarrow (\forall x \in U)[x \in A \wedge x \notin B]$
- Relationship between membership and subset: $(\forall x \in U)[x \in A \leftrightarrow \{x\} \subseteq A]$
- Definition of set equality: $A = B \leftrightarrow A \subseteq B \wedge B \subseteq A$

Do these represent the same sets or not?

$$X = \{x \in \mathbb{Z} | (\exists p \in \mathbb{Z})[x = 2p]\}$$
$$Y = \{y \in \mathbb{Z} | (\exists q \in \mathbb{Z})[y = 2q - 2]\}$$

$$A = \{x \in \mathbb{Z} | (\exists i \in \mathbb{Z})[x = 2i + 1]\}$$
$$B = \{x \in \mathbb{Z} | (\exists i \in \mathbb{Z})[x = 3i + 1]\}$$
$$A = \{x \in \mathbb{Z} | (\exists i \in \mathbb{Z})[x = 4i + 1]\}$$