CMSC 250
Discrete Structures

Set Theory

Proving two sets are equal using the rule sheet

<u>Claim:</u> A = B.

<u>Proof:</u>

A = X [By rule called…]

= Y [By rule called…]

= Z [By rule called…]

= B [By rule called…]

Deriving new properties using rules (or from definitions)

$$B - (A \bigcap C) = (B - A) \bigcup (B - C)$$

$$A - B = A - (A \bigcap B)$$

Using Venn diagrams to help find counterexamples

$$A \cup (B \cap C) = ? = (A \cap B) \cup (A \cap C)$$

$$A \cup (B - C) = ? = (A \cup B) - C$$

Trick: Draw the Venn diagrams and find a cell where they disagree. Make sure your counterexample has an element in that cell.

Proofs about power sets

- Claim: $A \subseteq B \rightarrow \mathcal{P}(A) \subseteq \mathcal{P}(B)$

- Claim: For finite sets, A: $[n(A) = k \rightarrow n(\mathcal{P}(A)) = 2^k]$

  [Think about inductive step with a small example.]

- Claim: $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$

  Does this work for union?

Partitions of a set

- A collection of nonempty sets $\{A_1, A_2, \ldots, A_n\}$ is a *partition* of the set A if and only if

  1. $A = A_1 \cup A_2 \cup \ldots \cup A_n$
  2. $A_1, A_2, \ldots, A_n$ are mutually disjoint

Russell's Paradox

Definition

Most sets are not elements of themselves.
Let S be the set of all sets that are not elements of themselves:

$$S = \{A | A \text{ is a set and } A \notin A\}$$

Is S an element of itself.