CMSC 250
Discrete Structures
Set Theory
Proving two sets are equal using the rule sheet

Claim: $A = B$.

Proof:

- $A = X$ [By rule called…]
- $= Y$ [By rule called…]
- $= Z$ [By rule called…]
- $= B$ [By rule called…]
Deriving new properties using rules (or from definitions)

\[ B - (A \cap C) = (B - A) \cup (B - C) \]

\[ A - B = A - (A \cap B) \]
Using Venn diagrams to help find counterexamples

\[ A \cup (B \cap C) = ? = (A \cap B) \cup (A \cap C) \]

\[ A \cup (B - C) = ? = (A \cup B) - C \]

Trick: Draw the Venn diagrams and find a cell where they disagree. Make sure your counterexample has an element in that cell.
Proofs about power sets

• Claim: \( A \subseteq B \implies \mathcal{P}(A) \subseteq \mathcal{P}(B) \)

• Claim: For finite sets, \( A: \ [n(A) = k \implies n(\mathcal{P}(A)) = 2^k] \)
  [Think about inductive step with a small example.]

• Claim: \( \mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B) \)
  Does this work for union?
Partitions of a set

- A collection of nonempty sets \( \{A_1, A_2, \ldots, A_n\} \) is a partition of the set \( A \) if and only if
  1. \( A = A_1 \cup A_2 \cup \ldots \cup A_n \)
  2. \( A_1, A_2, \ldots, A_n \) are mutually disjoint

An infinite set can be partitioned. The partitions can be infinite, or can be finite.

Examples
Russell’s Paradox

**Definition**

Most sets are not elements of themselves. Let $S$ be the set of all sets that are not elements of themselves:

$$S = \{ A | A \text{ is a set and } A \notin A \}$$

Is $S$ an element of itself.