$\{1, 2, 3, 4\}$

Repetition is not allowed and order doesn't matter

Subsets of 3 elements

$\{1, 1, 2\}$ X

$\{1, 2, 1\} \equiv \{2, 1, 1\}$

equivalent

order doesn't matter

$$\binom{n}{r} = \binom{4}{3} = \frac{4!}{3!\,1!}$$

$$= 4$$

$\{1, 2, 3, 4\}$

Find subsets of 3 elements, when order does not matter and repetition is allowed.

$\{1, 1, 1\}, \{1, 1, 2\}, \{1, 1, 3\}, \{1, 1, 4\}$ – 4

$\{1, 2, 2\}, \{1, 2, 3\}, \{1, 2, 4\}$ – 3

$\{1, 3, 3\}, \{1, 3, 4\}, \{1, 4, 4\}$ – 3

$\{2, 2, 2\}, \{2, 2, 3\}, \{2, 4, 4\}$ – 3

$\{2, 3, 3\}, \{2, 3, 4\}, \{2, 2, 4\}$ – 3

$\{3, 3, 3\}, \{3, 3, 4\}, \{3, 4, 4\}$ 3

$\{4, 4, 4\}$ 1

20

$$\begin{array}{c|c|c|c} 1 & 2 & 3 & 4 \\ * * & & & \\ * * & * & & \end{array}$$

$$\begin{array}{l} 4 \quad n \\ +\ 3 \quad r \end{array}$$

$$\binom{6}{3}$$

$$1 \mid 2 \mid 3 \mid 4$$

$$1 \mid\mid 2 \mid 3 \quad 4$$

$$\binom{n+r-1}{r} = \binom{4+3-1}{3}$$

$$= \binom{6}{3}$$

---

n — $n-1$ bars

r — pick $n+r-1$

r

$r = 4$

S	M	K
* ***		
***	*	
**	*	*

$n = 3$

$$\binom{n+r-1}{r} = \binom{6}{4}$$

$$= \frac{6!}{3!\,2!} = \frac{6 \times 5 \times 4}{3!} = 2$$

$$\frac{6!}{2!\,4!} = \frac{6 \times 5}{2} = 15$$

$n = 6$

$r = 25$

$$\binom{6+25-1}{25}$$

$$= \binom{30}{25}$$

$n = 5$

$r =$

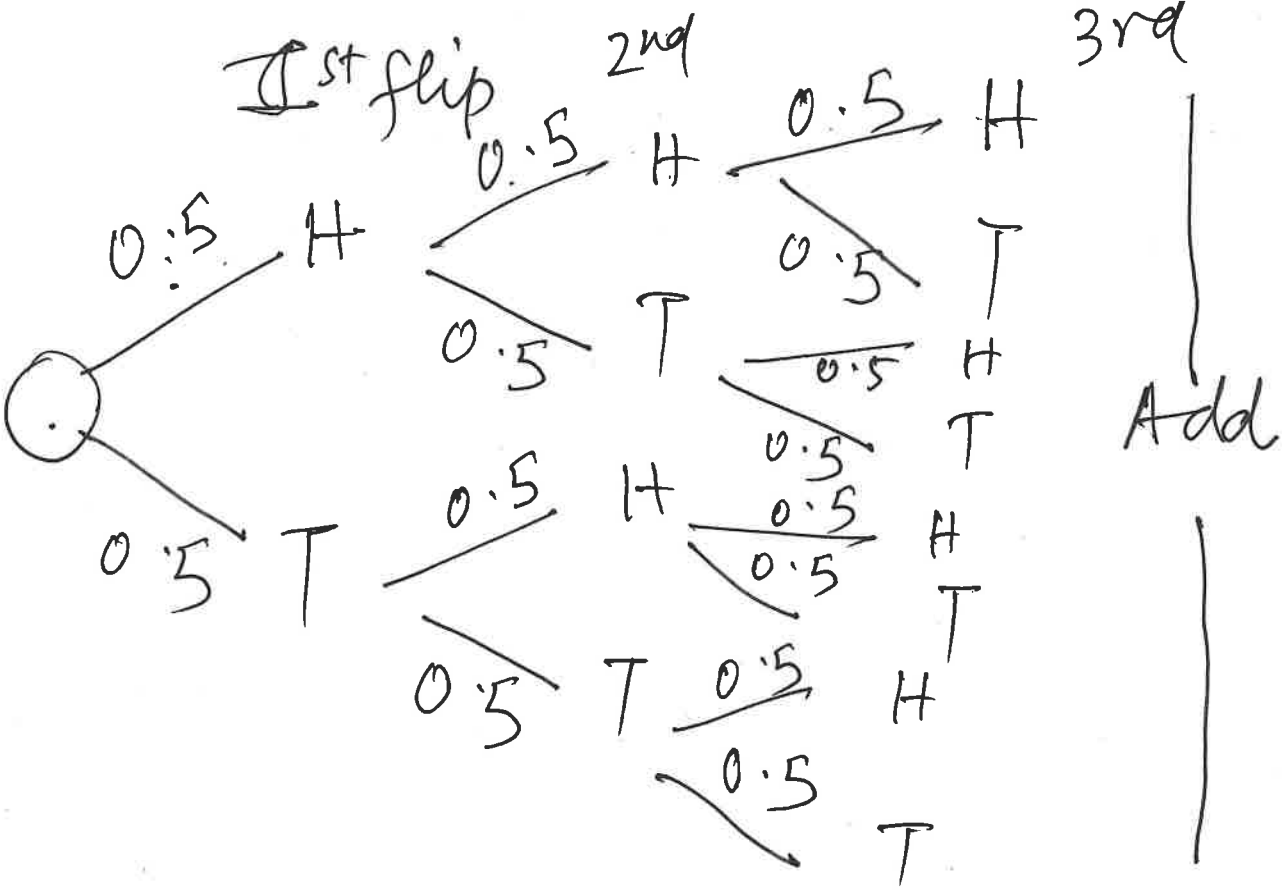
$$\binom{n+r-1}{r}$$

$$\binom{5+130-1}{130} = \binom{134}{130}$$

$n = 4$

$r = 50$

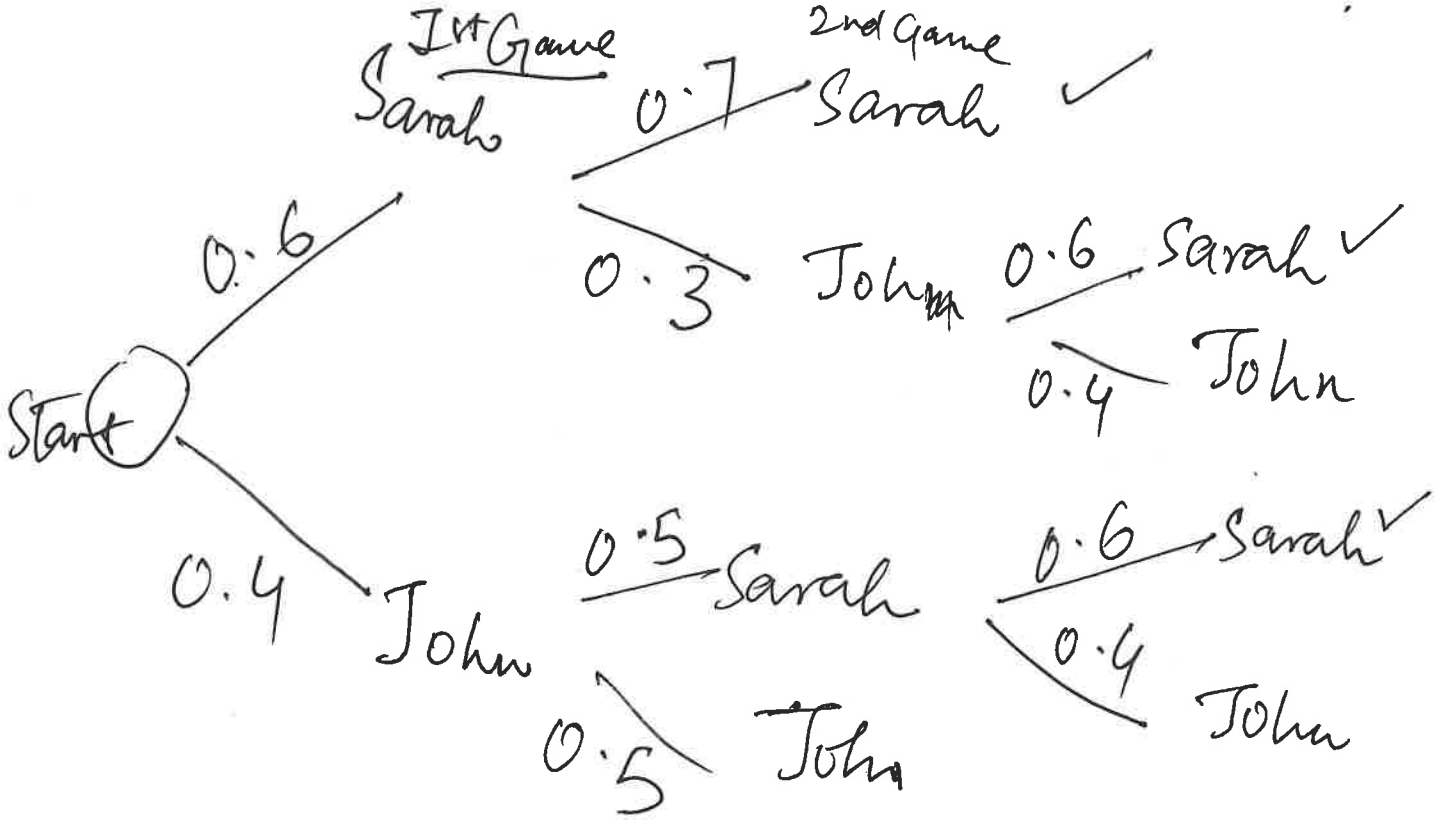
$$\binom{50+4-1}{50} = \binom{53}{50}$$



Multiply probabilities

$$P(3 \text{ heads}) = 0.5 \times 0.5 \times 0.5$$

$$= \frac{1}{8}$$



Sarah wins

$$(0.6 \times 0.7) + (0.6 \times 0.3 \times 0.6)$$

$$+ (0.42 \times 0.5 \times 0.6)$$

$$= 0.42 + 0.108 + 0.12$$

$$= 0.648$$

Tournament ends in two games

$$0.6 \times 0.7 + 0.4 \times 0.5 = 0.42 + 0.2$$

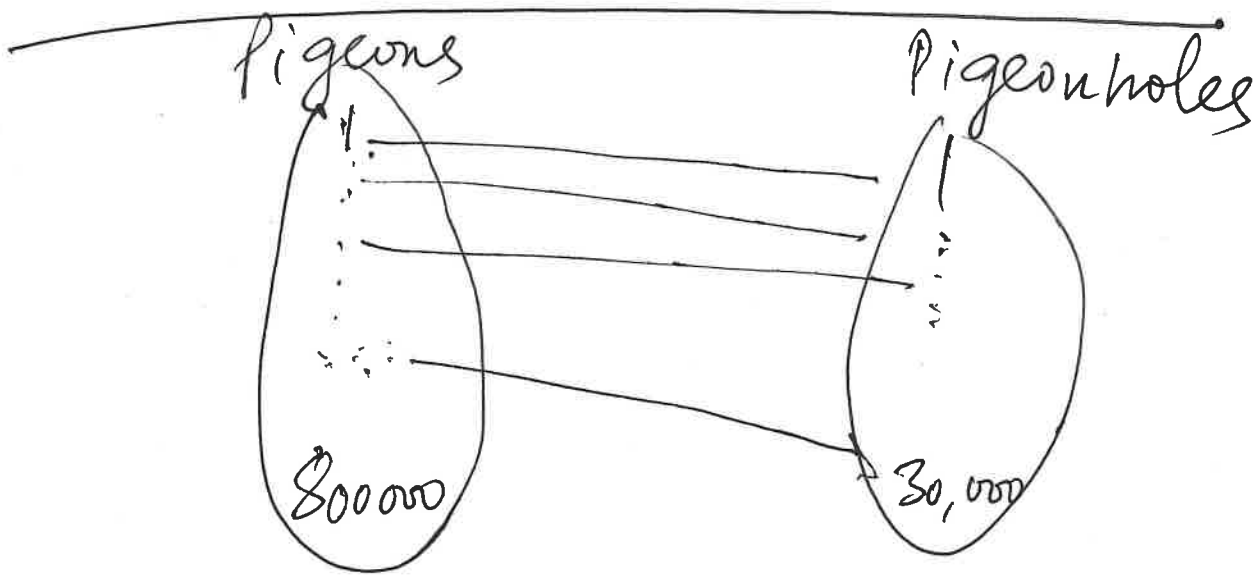
$$= 0.62$$

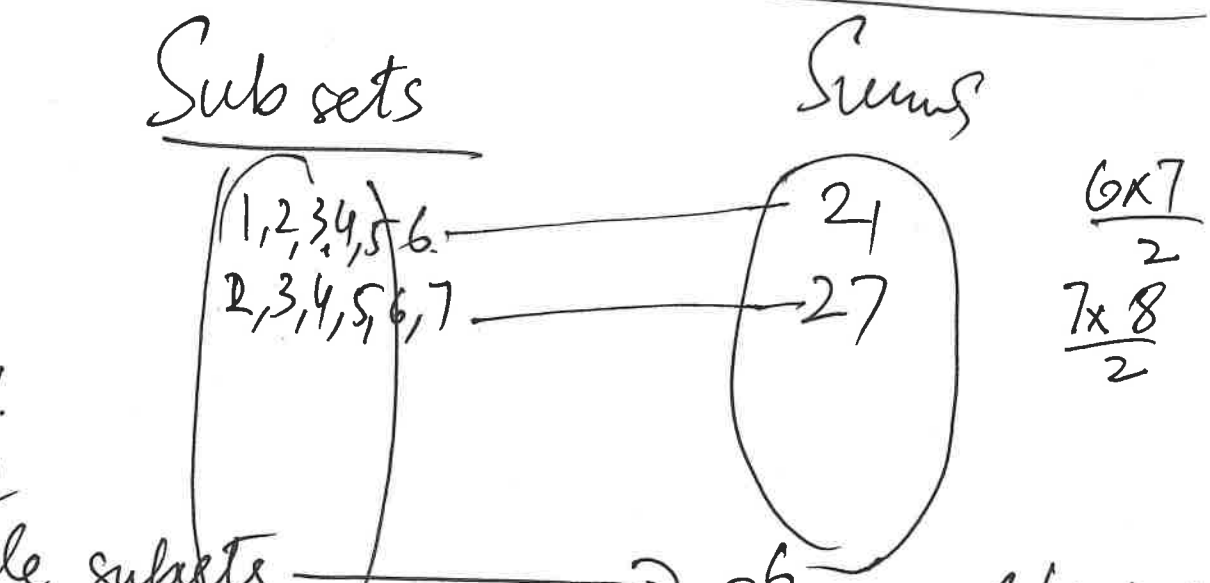
John wins in 3 games

$$(0.4 \times 0.5 \times 0.4) + (0.6 \times 0.3 \times 0.4)$$

$$= 0.08 + 0.072$$

$$= 0.152$$





$\frac{6 \times 7}{2}$

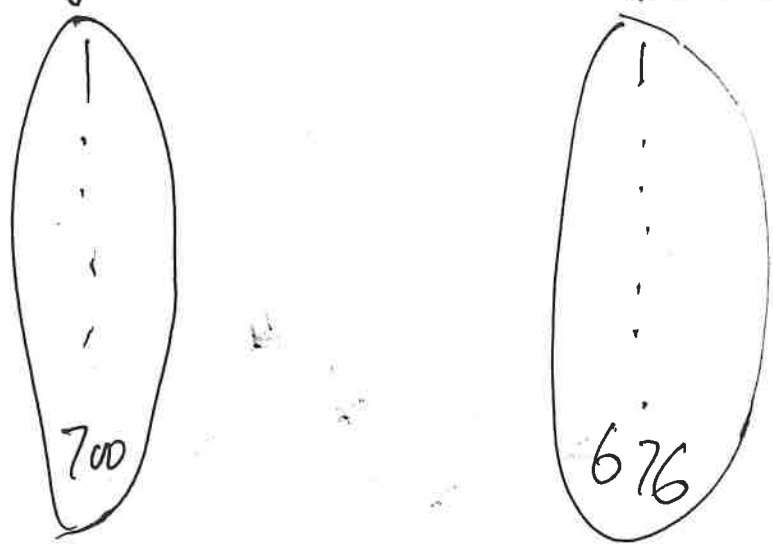
$\frac{7 \times 8}{2}$

Pigeons: # of possible subsets $\longrightarrow 2^6 = \underline{64 \text{ subsets}}$

Pigeons: # of sums ——

$\{5, 6, 7, 8, 9, 10\} \longrightarrow$ Sum 45

Pigeons | Pigeon holes



26×26

$676 < 700$

$2^{20} = 1048576$ pigeons

Sums largest for the subset

$$\sum_{i=981}^{1000} i = \sum_{i=1}^{1000} i - \sum_{i=1}^{980} i$$

$\{981, \ldots, 1000\}$