Summary: Choosing r elements out of n elements

	order matters	order doesn't matter
repetition allowed	$\underbrace{n \times \cdots \times n}_{r \text{ times}} = n^r$	$\binom{n+r-1}{r}$
repetition not allowed	$P(n,r) = \frac{n!}{(n-r)!}$	$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

Repeats allowed; order doesn't matter

Example

- I have a bag full of: 7 snickers bars, 12 milkyway bars, and 15 KitKats. How many ways are there to reach in and grab 4 pieces of candy?
- M&M's come in 6 colors. How many different handfuls of 25 M&M's are possible?

These are called **multisets**.

Repeats allowed; order doesn't matter

Number of multisets of size r taken from a set of size $n = \binom{n+r-1}{r}$

Example

- I have 130 students. How many grade distributions are possible? (For example, a grade distribution might be: 40 A's, 30 B's, 40 C's, 20 D's, and 10 F's.)
- How many ways are there to distribute 50 tennis balls among 4 containers. (Any number of balls could be put into any container, including 0.)?

Probability Tree

- Depicts scenario that happens in stages
- Makes it easy to answer almost any probability question about the outcome

Example:

John and Sarah are playing a chess tournament. They will play the best two out of three games.

- Sarah has a slight edge, so she has a 60% chance of winning the first game.
- If Sarah wins the first game, she gains confidence, so her chance of winning the second game is 70%
- If Sarah loses the first game, she loses confidence, so her chance of winning the second game is 50%
- The third game (if there is one) is back to 60% chance for Sarah.

Questions:

- What is the probability that Sarah wins the tournament?
- What is the probability that the tournament ends in two games?
- What is the probability that John wins, but it takes three games?