domain of $f = \{1, 3, 5, 9\}$

Co-domain of $f = \{2, 3, 5, 6, 7, 8\}$

range of $f = \{2, 7, 8\}$

$y = f(x)$

$y = x^2 + 1$ $\qquad x = 1$

y

$\nexists x$ such that $f(x) = y_1$

$f(x) = y_2$

range of f is also the codomain then that function f, is surjective or onto.

inverse of $A = \{1, 5, 2\}$

Inverse of $C = \{7, 12, 14\}$

inverse of $B = \phi$

$x_1, x_2, \ldots \quad x \in X \qquad y \in Y$

$y = f(x) \qquad x \in X$

Inverse of $y = x \qquad y \in Y$

For an injective function

$\forall x_1, x_2 \in X$ if $F(x_1) = F(x_2)$

then $\underline{x_1 = x_2}$

$a, b \in D$, $F(a) = F(b)$

if $F(a) = F(b)$ $\underline{\underline{a = b}}$ then

leads to the conclusion F is injective

$$f(x) = 3x + 7$$

Let us pick two values $a, b \in X$

such that $f(a) = 3a + 7$

$$f(b) = 3b + 7$$

Assume, $f(a) = f(b)$

$$3a + 7 = 3b + 7$$

Substract 7 from b/s

$$3a = 3b$$

divide by 3 on b/s

$\Rightarrow \quad a = b$

domain co-domain

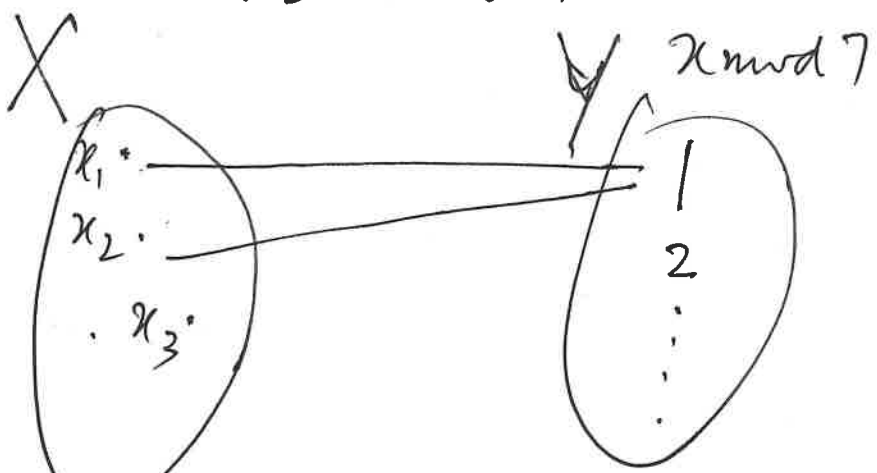
$f: \mathbb{Z} \to \mathbb{Z} \qquad f(x) = x \bmod 7$

let us pick $a, b \in X$ arbitrarily such that $f(a) = f(b)$

$\Rightarrow \quad a \bmod 7 = b \bmod 7$

$a = 15, \quad b = 22$

$15 \bmod 7 = 22 \bmod 7$



$x_1 = 15$

$x_2 = 22$

$$f : \mathbb{R} - \{1\} \to \mathbb{R}$$

$$f(x) = \frac{x+1}{x-1}$$

<u>Proof:</u>

Let $a, b \in D$ such that $f(a) = f(b)$

$$f(a) = \frac{a+1}{a-1}$$

$$f(b) = \frac{b+1}{b-1}$$

if $f(a) = f(b)$

$$\frac{a+1}{a-1} = \frac{b+1}{b-1}$$

$$(a+1)(b-1) = (b+1)(a-1)$$

$$\Rightarrow \quad ab - a + b - 1 = ab + a - b - 1$$

Subtract ab on b/s

$$-a + b - 1 = a - b - 1$$

Add 1 to both sides

$$-a + b = a - b$$

$$\cancel{b} + b = a + a$$

$$2b = 2a$$

$$\Rightarrow \quad a = b$$

Let $c \in C$ picked arbitrarily

$\exists\, d \in D$ such that $f(d) = c$

$$f(x) = 3x + 7$$

assign an arbitrary value

$$y = f(x)$$

$$y = 3x + 7$$

$$y - 7 = 3x$$

$$x = \frac{y-7}{3}$$

$y \in C$

$x \in D$

$f: \mathbb{R} \to \mathbb{Z} \qquad f(x) = \lfloor x/2 \rfloor$

let $c \in C$ be chosen arbitrarily

such

$$c = \lfloor d/2 \rfloor$$

where $d \in D$

$$c \leq \frac{d}{2} < c+1$$

$$d \geq 2c$$

<u>Surjective</u>

$f: \mathbb{R}^+ \to \mathbb{R}$ such that $f(x) = \sqrt{x}$

domain

$f(x)$

-5

codomain

$x = -5$

$\sqrt{-5}$

$f : X \to Y \qquad g : Y \to Z$

$(g \circ f) : X \to Z$

$(g \circ f)(x) = g(f(x))$

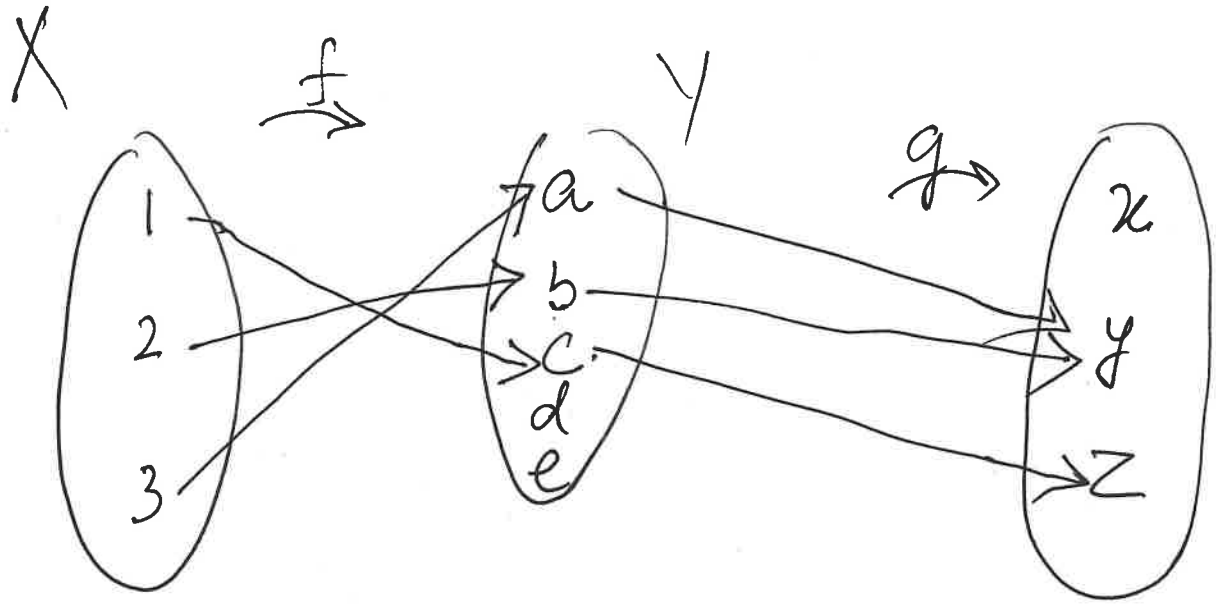
range of the first function is the domain of the second function.

f — first function

g — second function

$g \circ f$ — g circle f

$g(f(x))$ — g of f of x



$$(g \circ f)(1) = g(f(1))$$
$$= g(c) = z$$

$$(g \circ f)(2) = g(f(2)) = g(b) = y$$

$$(g \circ f)(3) = g(f(3)) = g(a) = y$$

$$(g \circ f)(x) = g(f(x))$$
$$= g(x+1)$$
$$= (x+1)^2$$

$$(f \circ g)(x) = f(g(x))$$

$$= f(x^2)$$

$$= x^2 + 1$$

$$(x+1)^2 \neq x^2 + 1$$

$$\therefore (g \circ f)(x) \neq (f \circ g)(x)$$