$$a_n = a_{n-1} + n - 1 \qquad a_1 = 0$$

$n = 5$

$$a_5 = \underline{a_4} + 4$$

$$= a_3 + (3 - 1) + 4$$

$$= a_3 + 3 + 4$$

$$= a_2 + 2 + 3 + 4$$

$$= a_1 + 1 + 2 + 3 + 4$$

$$= \sum_{i=0}^{n} i$$

$$\sum_{i=1}^{n} \prod_{j=1}^{i} 2$$

$$\sum_{i=1}^{n} 2^i$$

$$a \equiv_4 b$$

$$4 \mid a - b$$

Reflexive $\quad aRa$

$a \equiv_4 a = 4 \mid a - a = 4 \mid 0$

Every number including 4 divides 0

therefore $a \equiv_4 b$ is reflexive

Symmetric $\quad aRb \longrightarrow bRa$

if aRb then bRa

$4 \mid a - b$

$\Rightarrow \quad a - b = 4k$

Multiply b/s by -1

$b - a = 4(-k)$

$\Rightarrow \quad 4 \mid b - a$

Here $a \equiv_4 b$ and $b \equiv_4 a$

so it is symmetric

Transitive

if $aRb \wedge bRc \rightarrow aRc$

$4 \mid a-b$

$a-b=4k$

$b-c=4l$

Adding the two equations

$a-c=4(k+l)$ $\quad k, l \in \mathbb{Z}$

$\Rightarrow \quad 4 \mid a-c$

Therefore it is transitive and hence equivalent

$\frac{1}{2}, \frac{2}{4}, \frac{3}{6} \ldots$

Partition

Subsets A_1, A_2, A_3 are partitions of set A such that they are mutually disjoint and $A_1 \cup A_2 \cup A_3 = A$

$$A = \{1, 2, 3, 4, 5, 6\}$$

$$A_1 = \{1, 2\}, A_2 \{3, 4, 5\}, A_3 = \{6\}$$

$$A_3 = \{1, 6\} \quad \times$$

$R = \{$

aRa

There is a loop from every letter itself.

Symmetric

aRd and dRa

$bRf \wedge fRb$

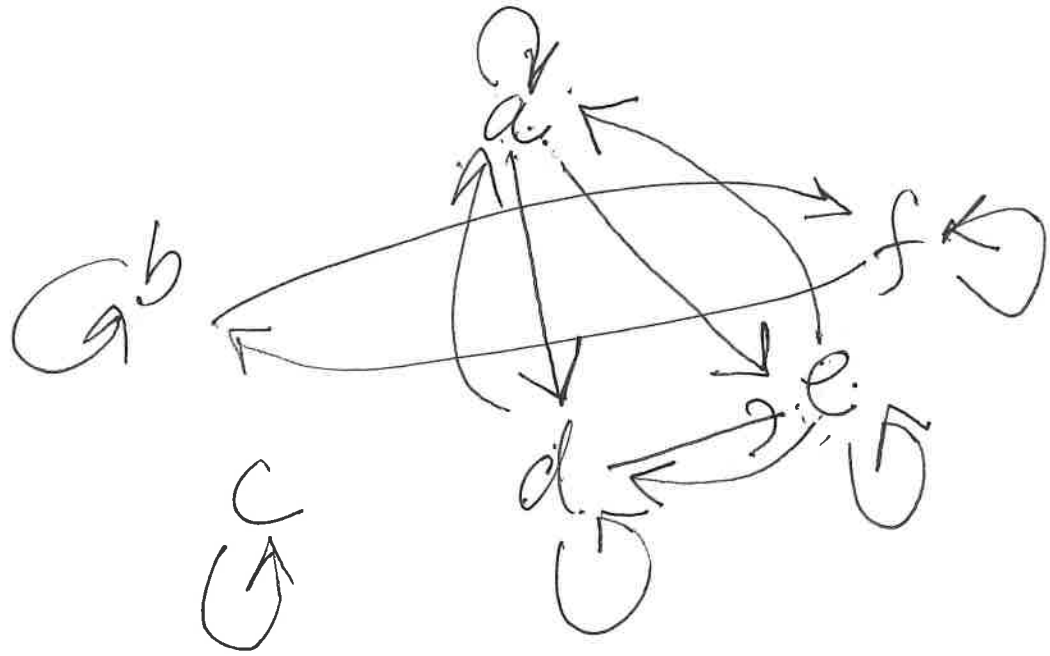
$dRe \wedge eRd$

$aRe \wedge eRa$

$aRd \wedge dRe \rightarrow aRe$

There is an arrow going from each one of these letters.

$X = \{a, b, c, d, e, f\}$



$xRy \quad x=y$

if $\overset{X}{aRb} \rightarrow bRa$

a	b	$a \rightarrow b$
T	T	T
T	F	F
F	T	T
F	F	T

Reflexive

Symmetric

$aRb \wedge bRc \rightarrow aRc$

$R = \{ (\text{Alabama}, \text{Arizona}) \ldots \}$

$\underline{a} \; R \; a$

Alabama R Alabama

Alabama R Arizona

then Arizona R Alabama

$\{ \; aRb \wedge bRc \rightarrow aRc$

$\sin(a) = \sin(b)$

Reflexive

$\sin(a) = \sin(a)$

Symmetric $\sin(a) = \sin(b)$

Also $\sin(b) = \sin(a)$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 \end{array}$$

$R = \{ (1,1), (2,1), (1,2) \}$

Symmetric
not antisymmetric

$R = \{ (1,1), (1,2), (1,3) \}$

Antisymmetric not symmetric

$R = \{ (1,1), (2,2), (3,3) \}$

Both symmetric and anti symmetric

$R = \{ (1,1), (2,2), (1,2), (2,1), (1,3) \}$

Neither symmetric nor anti symmetric

if aRb then bRa

aRb	$\rightarrow$	bRa
T	T	T
T	F	F
F	T	T
F	F	T

If $\sin(a) = \sin(b)$

$\sin(b) = \sin(c)$

then $\sin(a) = \sin(c)$

By the transitive property of equality.

Therefore it is equivalent.

Antisymmetric

$$\left[\begin{array}{l} aRb \wedge bRa \rightarrow a = b \\ \text{if } \underline{a \neq b} \quad \underline{aRb} \rightarrow b \not R a \end{array}\right]$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$\geq$ over $\mathbb{Z}$

$a \geq b$

Reflexive $a \geq a$ —— Yes.

$a \geq b \rightarrow b \ngeq a$

If $2 \geq 1$ then $1 \ngeq 2$

By the property of the greater than for integers. it is antisymmetric.

$a \geq b$ and $b \geq c \rightarrow a \geq c$

By the transitive property of inequality for integers.

$A = \{1, 3, 5, 6, 9\}$

$aRb \quad a|b$

$R = \{(1,1), (1,3), (3,6), (3,9), (3,3)\}$

Reflexive

$a | a$

Every integer divides itself.

~~Sy~~ Antisymmetric

$a | b \rightarrow b | a$

if $a | b$

$b = ak$

$a = b.l$

$a = akl \quad \Rightarrow kl = 1$

$aRb \wedge bRa \rightarrow a = b$

Transitive

$a \mid b$ and $b \mid c \rightarrow a \mid c$

$b = ak$

$c = bl$

$c = akl$.

Integers are closed in multiplication

$a \mid c$

Therefore, it is reflexive, antisymmetric and transitive.

Total —— Every element in relation / set is comparable to every other element.

$a \leq b$ or $b \leq a$

Transitive

$a \leq b$ and $a \neq b \rightarrow b \not\leq a$

Every total order relationship is a partial order relation by every partial order relation is not a total order relation.

light lighthouse lightning