Recall: Properties of binary relations

Reflexive	$(\forall a \in A)\ [aRa]$
Irreflexive	$(\forall a \in A)\ [a \not R a]$
Symmetric	$(\forall a, b \in A)\ [aRb \rightarrow bRa]$
Antisymmetric	$(\forall a, b \in A)\ [aRb \wedge bRa \rightarrow a = b]$
Asymmetric	$(\forall a, b \in A)\ [aRb \rightarrow b \not R a]$
Non-symmetric	$(\forall a, b \in A)\ [a \neq b \rightarrow (aRb \leftrightarrow b \not R a)]$
Transitive	$(\forall a, b, c \in A)\ [aRb \wedge bRc \rightarrow aRc]$

Equivalence relations

- A binary relation is an **equivalence relation** iff it is:
  - reflexive,
  - symmetric, and
  - Transitive

Example:

Let R be the relation over $\mathbb{Z}\times\mathbb{Z}$ defined by:

aRb iff $a \equiv_4 b$

(Let's verify that this is an equivalence relation.)

Equivalence relations

- An equivalence relation forms a *partition* of the elements:
  All elements that are related to one another are within the same partition.
- These partitions are called equivalence classes
  - [a] = the equivalence class containing a
  - [a] = $\{x \in A \mid xRa\}$

More Equivalence Relations

- Let X = {a,b,c,d,e,f}, and define the following binary relation over X:

  R = {(a,a),(b,b),(c,c),(d,d),(e,e),(f,f),(a,e),(a,d),(d,a),(d,e),
  (e,a),(e,d),(b,f),(f,b)}

- Let R be a binary relation defined over the 50 states in the U.S. as:

  aRb iff the names of a and b start with the same letter

- Let R be a binary relation over $\mathbb{R}$ defined by:

  aRb iff sin(a) = sin(b)

- Let f be *any* function with domain D. Define a binary relation R over D as:

  aRb iff f(a)=f(b)

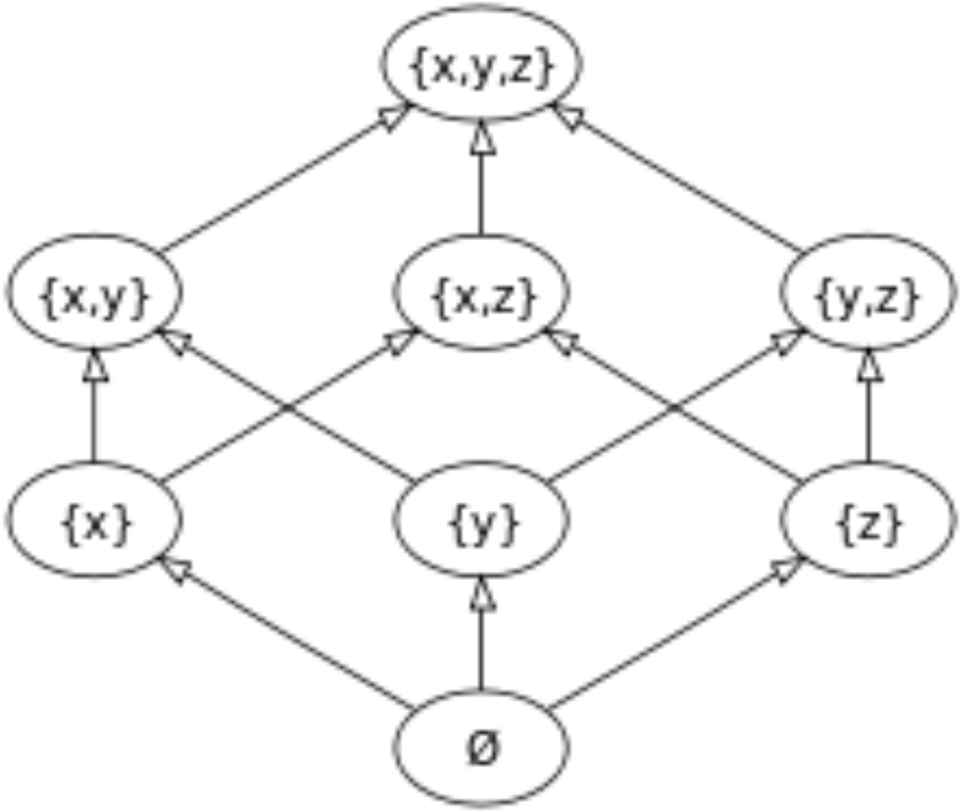
Partial order relation

- R is a **partial order relation** if and only if
  R is reflexive, antisymmetric, and transitive

- Examples
  - $\geq$ over $\mathbb{Z}$
  - divisibility over $\mathbb{Z}^+$
  - $\subseteq$ over any collection of sets

Partial order relation

- Partial orders correspond to "reachability" in *directed acyclic graphs* (DAGs)



Total ordering

A relation, R, is **total** (over S) if for all elements a, b ∈ S:

aRb or bRa

A relation is a **total order** relation if it is:

- Total
- Transitive
- Antisymmetric

Examples:

- ≤ over $\mathbb{R}$
- Lexicographical ordering of English words