CMSC 250 Homework #1, Due: Tuesday, February 12

This assignment is due before midnight (11:59 PM) on the due date. Please note that there are a total of seven numbered questions. Be sure to write legibly or we will not be able to grade your answer. You do not have to answer using complete sentences, and you do not need to copy down the questions – we know what the questions are.

1. [15 pts.] Let m, a, n, o, f, and i mean the following:
   - m = "Jack Bauer was awake in the morning (from 4:00AM to 12:00PM)."
   - a = "Jack Bauer was awake in the afternoon (from 12:00PM to 8:00PM)."
   - n = "Jack Bauer was awake in the night (from 8:00PM to 4:00AM)."
   - o = "Tony Almeida was awake in the morning."
   - f = "Tony Almeida was awake in the afternoon."
   - i = "Tony Almeida was awake in the night."

   Assume that the universe of discourse (the set of possibilities under discussion) is a particular 24 hour period. Give logical expressions, using the propositions above, which mean the following:

   a. Jack slept during the entire 24 hours (all three time intervals).
   b. Jack was awake for at least 8 hours.
   c. Jack was asleep for at least 8 hours.
   d. At any time, at least one of the two men were awake.
   e. Jack and Tony were always awake at the same time.

2. [14 pts.] For each of the following say if it is is true (T), false (F), can be assigned a truth value but most people do not know what that value is (S) [S means "statement"], or cannot be assigned a truth value (NS) [NS means "not a statement"].

   a. The CSIC building is made from exactly 427,379 bricks.
   b. $2^4 = 4^2$
   c. $x = 7$
   d. Carrots taste better than celery.
   e. What is the meaning of life, the universe, and everything?
   f. My calculus book was once the president of the United States.
   g. There is life in the Andromeda galaxy or there is not life in the Andromeda galaxy.

3. [9 pts] For each of the following statements, give its converse, its inverse, and its contrapositive.

   Be sure to clearly label which one is the converse, which one is the inverse, and which one is the contrapositive. When negating statements, you may not simply prepend the statement with "It is not the case that..." or "it is false that...", and you must avoid using the phrases "not every", "not always", 'and 'not all".

   a. If you are always late then you do not have a watch.
   b. If all men are mortal then I will die one day.
   c. If none of the NFL teams are cheaters then all footballs are properly inflated.

4. [13 pts.]

   This question will require you to draw a truth table. Please be sure to use the same style and pattern for entering truth valuations that was demonstrated in class:

   - The columns for your variables must appear in alphabetical order.
   - Use 1 for true and 0 for false.
   - Fill in the truth values for the variable columns using the exact pattern illustrated below:

p	q	r
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

   Although this convention is arbitrary, we require it to make things easier for the graders. You will lose points for not using this style on this question and any others during the course that require truth tables.

   Using a truth table, decide whether or not the statements $(p \to q) \to r$ and $p \to (q \to r)$ are logically equivalent. After drawing the truth table, state whether or not they are logically equivalent and tell us what aspect of the truth table tells you this.

5. [14 pts.] For this question, you must use the Equivalency Law Table (available on the class webpage) along with the two rules below:

$$\begin{aligned} p \to q &\equiv \sim p \vee q && \text{definition of } \to \\ p \leftrightarrow q &\equiv (p \to q) \wedge (q \to p) && \text{definition of } \leftrightarrow \end{aligned}$$

   For each pair of statements below, show that the two statements are logically equivalent. Use the format of derivation that was demonstrated in class– you must justify each line using one of the rules, you may not skip any steps at all, and you may not combine two steps on one line!

   a. $p \wedge q \to r$ and $\sim p \vee (q \to r)$

   b. $p \leftrightarrow \sim q$ and $(\sim p \wedge q) \vee (p \wedge \sim q)$

6. [14 pts.] Construct a complete truth table to help you determine if the following argument is valid or not. State whether it is valid or not, indicate the entries in the truth table that led you to your answer, and explain why those entries support your answer.

$$\begin{array}{ll} \text{P1:} & \sim p \\ \text{P2:} & \sim(r \to q) \\ \hline \therefore & \sim(r \to (p \vee q)) \end{array}$$

7. [21 pts.]Prove the validity of the following arguments using the Laws of Equivalency (including definition of $\rightarrow$ and definition of $\leftrightarrow$) along with the Rules of Inference that were shown in class. (Both the Laws of Equivalency and the Rules of Inference are available on the class webpage.)

a. P1 $\sim a \rightarrow b$
P2 $s \rightarrow t$
P3 $a \rightarrow s$
P4 $b \rightarrow s$
$\therefore$ t

b. P1 $\sim(p \rightarrow q)$
P2 $\sim p \vee s$
$\therefore$ $\sim(s \rightarrow q)$

c. P1 $(y \rightarrow r) \rightarrow t$
P2 $y \rightarrow z$
P3 $\sim z$
$\therefore$ t