CMSC 250 Homework #2, Due: Wednesday, February 20

Please note that there are a total of six numbered questions. You do not have to answer using complete sentences, and you do not need to copy down the questions – we know what the questions are. When submitting on Gradescope, be sure to complete the step where you identify which page contains each question – if you skip this step your grade on the assignment will be 0.

1. [17 pts]

   Consider the following truth table:

p	q	r	output
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

   a. [6 pts] Give a statement in propositional logic that corresponds to the last column.

   b. [11 pts] Draw a circuit that corresponds to this truth table. (You should have one input for each variable, and just a single output.)

2. [9 pts] Consider constructing a circuit for an "adder" (as shown in class) that will add together two 64-bit operands.

   a. How many output bits would there be?

   b. How many "half adders" would be used?

   c. How many "full adders" would be used?

3. [8 pts] Perform the following number conversions:

   a. Write 1100011_2 in base 10.

   b. Write 181_{10} in binary.

4. [24 pts] Let the predicate $L(p, f)$ mean "person p likes eating food f". Let P refer to the domain of "all people" and let F refer to the domain of "all food". Translate each of the following English sentences into a statement of predicate logic.

   a. There is a food that someone likes.

   b. There is someone who likes all foods.

   c. Each food is liked by some people.

   d. There is no food that everyone likes.

   e. There is a food that everyone dislikes.

   f. Everyone dislikes every food.

   g. There is a person who likes exactly one food.

   h. There is a person who likes at least two foods.

5. [24 pts] For each of the following statements:

   - Find a *finite but non-empty* domain where it is true, or if no such domain exists, write the word "IMPOSSIBLE".
   - Find an *infinite* domain where it is true, or if no such domain exists, write the word "IMPOSSIBLE".

   Your domains must contain only numbers, and we are assuming that the less than symbol ($<$) is being interpreted in the usual way.

   a. $(\forall x)(\forall y)[x = y]$
   b. $(\forall x)(\exists y)[x < y]$
   c. $(\exists y)(\forall x)[x < y]$
   d. $(\forall x)(\forall y)(\exists z)[x < y \ \rightarrow \ x < z < y]$
   e. $(\forall x)(\exists y)[y^2 = x]$
   f. $(\exists x)(\exists y)(\forall z)[x < y \ \wedge \ x \leq z \leq y]$

6. [18 pts] Recall that an "interpretation" in predicate logic consists of two things:

   - Specifying a domain.
   - Defining what the predicate symbol(s) mean.

   For each of the following statements give an interpretation that makes the statement true, and give an interpretation that makes the statement false.

   a. $(\forall x)\,(\forall y)\,(\forall z)\ [P(x, y) \wedge P(y, z) \rightarrow P(x, z)]$
   b. $(\forall x)\,(\forall y)\ [P(x, y) \rightarrow P(y, x)]$
   c. $(\forall x)\,P(x, x)$