1. Evaluate the following expressions:

(a) \[ \sum_{i=-2}^{2}(i^2 + 1) \]

(b) \[ \sum_{i=1}^{3} \sum_{j=1}^{3}(3i + j) \]

(c) \[ \prod_{i=1}^{3} \sum_{j=1}^{3}(ij) \]

2. For each of the following claims:

- Re-state the claim using summation notation, if applicable.
- Prove the claim by induction. Be sure to carefully show all of the following steps:
  - Assert that you are inducting on a particular variable.
  - State the element for which the base case applies, and prove it.
  - State the inductive hypothesis.
  - Label the inductive step and state what you must show.
  - Prove the inductive step, being careful to label the point at which the inductive hypothesis is being applied.

(a) Claim: For all \( n > 0 \): \( 2 + 4 + 6 + \ldots + 2n = n^2 + n \)

(b) Claim: For all \( n \geq 3 \): \( 4^3 + 4^4 + 4^5 + \ldots + 4^n = 4 \left( \frac{4^n - 16}{3} \right) \)

(c) Claim: For all \( n \geq 1 \): \( 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \)

(d) Claim: For all \( n \geq 0 \): \( 6|7^n - 1 \)

(e) Claim: For all \( n \geq 4 \): \( 2^n < n! \)

(f) Claim: For all \( n \geq 1 \): \( \sum_{i=1}^{n} \frac{1}{i^2} \leq 2 \)

Hint: This proof is much easier if you first prove that (for all \( n \geq 1 \)): \( \sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n} \)