1. Evaluate the following expressions:

   (a) $\sum\limits_{i=-2}^{2} \left(i^2 + 1\right)$

   (b) $\sum\limits_{i=1}^{3} \sum\limits_{j=1}^{3} \left(3i + j\right)$

   (c) $\prod\limits_{i=1}^{3} \sum\limits_{j=1}^{3} \left(ij\right)$

2. For each of the following claims:

   - Re-state the claim using summation notation, if applicable.
   - Prove the claim by induction. Be sure to carefully show all of the following steps:
     - Assert that you are inducting on a particular variable.
     - State the element for which the base case applies, and prove it.
     - State the inductive hypothesis.
     - Label the inductive step and state what you must show.
     - Prove the inductive step, being careful to label the point at which the inductive hypothesis is being applied.

   (a) Claim: For all $n > 0 : 2 + 4 + 6 + \ldots + 2n = n^2 + n$

   (b) Claim: For all $n \geq 3 : 4^3 + 4^4 + 4^5 + \ldots + 4^n = 4\left(\frac{4^n - 16}{3}\right)$

   (c) Claim: For all $n \geq 1 : 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

   (d) Claim: For all $n \geq 0 : 6 | 7^n - 1$

   (e) Claim: For all $n \geq 4 : 2^n < n!$

   (f) Claim: For all $n \geq 1 : \sum\limits_{i=1}^{n} \frac{1}{i^2} \leq 2$

   Hint: This proof is much easier if you first prove that (for all $n \geq 1$): $\sum\limits_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}$