1. Prove each of the following claims using strong induction. Be sure to:

   - Assert that you are invoking strong induction on a particular variable.
   - State the element(s) for which the base case(s) apply, and prove them.
   - Carefully state the inductive hypothesis. (Be sure to follow the examples from class!)
   - Label the inductive step and state what you must show
   - Prove the inductive step, being careful to label the point at which the inductive hypothesis is being applied.

   (a) Let a_n be the recurrence defined by: $a_0 = 12, a_1 = 29$,
   and $\forall n \geq 2, a_n = 5a_{n-1} - 6a_{n-2}$.
   Prove that $\forall n \geq 0 : a_n = 5 \cdot 3^n + 7 \cdot 2^n$

   (b) Let a_n be the recurrence defined by: $a_0 = 1, a_1 = 2, a_2 = 3$, and,
   $\forall n \geq 3, a_n = a_{n-1} + a_{n-2} + a_{n-3}$.
   Prove that $\forall n \geq 0 : a_n \leq 3^n$.

   (c) Let a_n be the recurrence defined by: $a_1 = 1, a_2 = 3$, and,
   $\forall n \geq 3, a_n = a_{n-1} + a_{n-2}$.
   Prove that $\forall n \geq 1 : a_n \leq \left(\frac{7}{4}\right)^n$.

   (d) Prove that if you only have 3 cent and 5 cent coins available, you can generate any amount of money that is greater than or equal to 13 cents.

2. Let a_n be the recurrence defined by: $a_0 = 4, a_1 = 7$,
   and $\forall n \geq 2, a_n = 2a_{n-1} + 5a_{n-2}$.
   Using constructive induction, find integer constants A and B such that,
   $\forall n \geq 0 : a_n \leq AB^n$.
   Try to make B as small as possible.