CMSC 250 Homework #7, Due Thursday 4/25

1. Prove each statement is true or find a specific counterexample. Assume all sets are subsets of a universal set, U. If the statement is true, prove it TWO different ways: First, straight from the definitions (i.e.: let x be an arbitrary element of some set, etc). Then prove it using the rules of set theory on your sheet of rules and equivalencies.

   a. For all sets A, B, and C, $(B \cup C) - A = (B - A) \cup (C - A)$.

   b. For all sets A, B, and C, $(A \cap C) - (C \cup A) = \emptyset$

   c. For all sets A, B, and C, $(A \cap B) \cap C = A - (C^c \cup B^c)$

   d. For all sets A, B, and C, $((A \cup B) - C) \cup (A \cap B) = ((A - B) \cup (B - A)) - C$

2. Prove each statement is true or find a specific counterexample. Assume all sets are subsets of a universal set, U.

   a. For all sets A, B, and C, $A \times (B \cup C) = (A \times B) \cap (A \times C)$.

   b. For all sets A and B, $|\mathcal{P}(A \times B)| \neq |\mathcal{P}(A) \times \mathcal{P}(B)|$.

   c. For all sets A and B, if $A - B = \emptyset$ then $A = B$.

   d. For all sets A, B, and C, $(A - B) \cup (B - A) \neq (A \cup B) - (A \cap B \cap C)$.

   e. If $A \cap B = A$, then $A \cup B = B$.

   f. If $A \cap B = A$ and $B \cap C = B$, then $A \cap C = A$.

   g. $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.