Homework 9 – Due Thursday 5/9

1. Decide whether or not each of the following functions is *injective*. Prove your answer formally.

   a. $f: \mathbb{N} \to \mathbb{R}$ such that $f(n) = 3/(n+5)$

   b. $f: \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ such that $f\big(\langle a, b \rangle\big) = a^b$

2. Decide whether or not each of the following functions is *surjective*. Prove your answer formally.

   a. $f: \mathbb{N} \to \mathbb{R}$ such that $f(n) = 3/(n+5)$

   b. $f: \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ such that $f\big(\langle a, b \rangle\big) = a^b$

3. Let $f(x) = x^2$. Note that I have not specified the domain or co-domain. For this problem domains and co-domains are subsets of the reals. (You do not have to prove anything in this question, just specify domains and co-domains as instructed below.)

   a. Give a domain and co-domain such that *f* is a bijection.

   b. Give a domain and co-domain such that *f* is injective, but not surjective.

   c. Give a domain and co-domain such that *f* is surjective, but not injective.

   d. Give a domain and co-domain such that *f* is neither injective nor surjective.

[There are more questions on the next pages.]

4. Below is the current roster for the Baltimore Orioles baseball team. They look pretty horrible this year. Please answer the questions on the following page.

Pitchers

#	Name	B/T	Ht	Wt	DOB
37	Dylan Bundy	S/R	6'1"	200lbs	11/15/92
54	Andrew Cashner	R/R	6'6"	235lbs	9/11/86
50	Miguel Castro	R/R	6'7"	205lbs	12/24/94
51	Paul Fry	L/L	6'0"	190lbs	7/26/92
60	Mychal Givens	R/R	6'0"	210lbs	5/13/90
41	David Hess	R/R	6'2"	180lbs	7/10/93
70	Josh Lucas	R/R	6'6"	185lbs	11/5/90
67	John Means	L/L	6'3"	230lbs	4/24/93
58	Evan Phillips	R/R	6'2"	215lbs	9/11/94
66	Tanner Scott	R/L	6'2"	220lbs	7/22/94
53	Dan Straily	R/R	6'2"	220lbs	12/1/88
43	Mike Wright	R/R	6'6"	215lbs	1/3/90
31	Jimmy Yacabonis	R/R	6'3"	205lbs	3/21/92

Catchers

#	Name	B/T	Ht	Wt	DOB
28	Pedro Severino	R/R	6'1"	219lbs	7/20/93
40	Jesus Sucre	R/R	6'0"	200lbs	4/30/88

Infield

#	Name	B/T	Ht	Wt	DOB
57	Hanser Alberto	R/R	5'11"	215lbs	10/17/92
19	Chris Davis	L/R	6'3"	230lbs	3/17/86
1	Richie Martin	R/R	5'11"	190lbs	12/22/94
39	Renato Nunez	R/R	6'1"	220lbs	4/4/94
14	Rio Ruiz	L/R	6'1"	215lbs	5/22/94
2	Jonathan Villar	S/R	6'1"	215lbs	5/2/91

Outfield

#	Name	B/T	Ht	Wt	DOB
16	Trey Mancini	R/R	6'4"	215lbs	3/18/92
3	Cedric Mullins	S/L	5'8"	175lbs	10/1/94
23	Joey Rickard	R/L	6'1"	185lbs	5/21/91
35	Dwight Smith Jr.	L/R	6'0"	210lbs	10/26/92

Please give exact numerical answers for all questions on this page. (Your final answer for each part should be just a single number!)

a. The Orioles manager needs to select 10 players for today's team. A baseball team must have:
   - A pitcher
   - A catcher
   - A left-fielder (this must be one of the outfielders from the roster)
   - A right-fielder (this must be one of the outfielders)
   - A center-fielder (this must be one of the outfielders)
   - A first-baseman (this must be one of the infielders)
   - A second-baseman (this must be one of the infielders)
   - A third-baseman (this must be one of the infielders)
   - A shortstop (this must be one of the infielders)
   - A designated hitter (this could be anyone -- even be a player who is listed under the "pitchers" section of the roster)

   How many choices does the manager have for today's team?

b. If today's team is chosen randomly (assume that each viable team from the previous question is equally likely), what is the probability that all of the players are at least 6 feet tall?

c. In the American League, the pitcher in the game does not bat. Each of the other 9 players on the team are put into a "batting order". (Somebody bats first, somebody bats second, etc.). Below is a typical batting order:
   1. Rickard (outfielder)
   2. Mancini (designated hitter)
   3. Mullins (outfielder)
   4. Smith (outfielder)
   5. Davis (infielder)
   6. Martin (infielder)
   7. Severino (catcher)
   8. Alberto (infielder)
   9. Nunez (infielder)

   How many different batting orders are possible for the Orioles?

   Hints:
   - You have to have 1 catcher, three outfielders, four infielders, and a designated hitter (who could be anyone, even those players who are listed under the "pitchers" section of the roster).

   - Don't rely on your answer from the first question because the batting order doesn't involve specific positions. In other words, if Rickard is batting fourth it doesn't matter whether he is playing left-field, center-field, or right-field.