CMSC 250
Discrete Structures

Spring 2019

Administration

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Administration (continued)

- Webpage
  - Get homework assignments
  - Syllabus
  - Other documents
- Piazza
  - Ask questions
    - Do **not** post solutions.
    - Do **not** ask if your answer or approach is correct.
  - Discuss issues
  - Public versus Private
- ELMS & Gradeserver
  - Get homework solutions
  - See grades
- Gradescope
  - Hand in homework
  - See graded homeworks and exams

Administration (continued)

- Textbook (bookstore/on reserve at McKeldin Library)
  - Susanna S. Epp, *Discrete Mathematics*. Cengage Learning. (Any edition is fine.)
- Homework
  - Regular homeworks: typically every week.
  - Must be in PDF.
  - Must be easy to read (your responsibility).
  - Late date: 20% off your actual grade.
  - Your neighbor should understand your answers.
  - Do problems from book (and other books).

Administration (continued)

- Class attendance
  - You are responsible for what is said in class.
- Office hours
- Grading
- Exams
  - Two midterms: **in lecture**.
    - Thursday, February 28th
    - Thursday, April 11th
  - Final exam: **4:00-6:00pm**.
    - Saturday, May 18th
- Academic integrity.

Topics (tentative)

- Prop Logic, Circuits, Pred Logic
- Techniques of Proof
- Number Theory
- Mathematical Induction
- Counting
- Functions, Relations, and Graphs

What is Discrete Mathematics?

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In contrast: *Continuous mathematics*

Discrete vs. Continuous Math: Examples

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Discrete Mathematics

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Discrete Mathematics

- Number theory (integers)
- Logic
- Combinatorics
- Graph theory
- Theory of computation

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Discrete vs. Continuous Math: Examples

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Continuous Mathematics

- Calculus (real numbers)
- Topology

Applications?

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- Directly applied
  Circuits to do addition
- Good to know
  Cannot store the digits of $\sqrt{2}$ on a computer.
- Interesting
  The number of primes is infinite.

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- Useful on the job.

What is Logic?

Definition

Logic is a science of reasoning or inference.

- Philosophy
- Mathematics
- Computer Science

Question

Why should computer scientists learn logic?

Smart Logic

Sherlock Holmes and Dr. Watson go on a camping trip. After a good dinner and a bottle of wine, they retire for the night, and go to sleep. Some hours later, Holmes wakes up and nudges his faithful friend. “Watson, look up at the sky and tell me what you see.” “I see millions and millions of stars, Holmes” replies Watson. “And what do you deduce from that?” Watson ponders for a minute. “Well, astronomically, it tells me that there are millions of galaxies and potentially billions of planets. Astrologically, I observe that Saturn is in Leo. Horologically, I deduce that the time is approximately a quarter past three. Meteorologically, I suspect that we will have a beautiful day tomorrow. Theologically, I can see that God is all powerful, and that we are a small and insignificant part of the universe. What does it tell you, Holmes?” Holmes is silent for a moment. "Watson, you idiot!” he says. “Someone has stolen our tent!”

What is Logic?

Definition

A *proposition* is a declarative statement that is either true or false.

Example

- January has thirty one days.
- $2 + 2 = 4$
- $2 + 2 = 5$
- What time is it?
- $x + 2 = 4$
- $x \times 0 = 0$

Operations on propositions

variables

A statement is represented by a letter variable, such as, p, q, r, s, etc.

Logical Connectives

- Conjunction (“AND”) - $\wedge$
- Disjunction (“OR”) - $\vee$
- Negation (“NOT”) - $\sim$ (or sometimes $\neg$)
- Exclusive OR (“XOR”) - $\oplus$

AND

Definition

The *AND* of two propositions, p and q, is the proposition "p and q". It is true when both p and q are true. It is denoted as $p \wedge q$.

Example

p: The sun is hot.

q: The moon is cold.

r: The sky is blue.

$p \wedge q$: The sun is hot and the moon is cold.

$q \wedge r$: The moon is cold and the sky is blue.

OR

Definition

The *OR* of two propositions, p and q, is the proposition "p or q". It is true when either p or q is true. It is denoted as $p \vee q$.

Example

p: The sun is hot.

q: The moon is cold.

r: The grass is blue.

$p \vee q$: The sun is hot or the moon is cold.

$q \vee r$: The moon is cold or the grass is blue.

NOT

Definition

The *NOT* of one proposition, p is the proposition "NOT p". It is true when p is false. It is denoted as $\neg p$.

Example

p: The sun is hot.

q: The moon is cold.

$\neg p$: The sun is not hot.

$\neg q$: The moon is not cold.

Truth tables (TT)

The meaning of a logical operation can be expressed as its "truth table"

Construct truth table for AND, OR, and NOT

XOR

The word "or" is often used to mean "one or the other", but this is not the same meaning of "or" in logic!

Definition

The *XOR* of two propositions, p and q is true when either p is true q is true, but not both. We denote it as $p \oplus q$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Precedence

Definition

Both OR and AND are associative, so $(p \vee q \vee r)$ is the same as $((p \vee q) \vee r)$, and as $(p \vee (q \vee r))$, and similarly $(p \wedge q \wedge r)$ is the same as $((p \wedge q) \wedge r)$ and as $(p \wedge (q \wedge r))$

AND and OR have equal precedence, so $(p \wedge q \vee r)$ is ambiguous without parentheses.
AND takes precedence when written multiplicatively (as in $pq \vee qr$ for $(p \wedge q) \vee (q \wedge r)$.

Precedence

Definition

A variety of English words translate into logic as $\wedge$, $\vee$, or $\sim$ $(\neg)$

Example

- *but* translates the same as *and* when it links two independent statements
  Banana is yellow but it is not sweet.
- *neither-nor* mean two negatives connected by *and*.
  Neither a borrower nor a lender be, may be represented as neither p nor q
  $\sim p$ and $\sim q$.

Practice Translating

- I am hungry or I am tired.
- Bob was tall and thin.
- Apples are healthy but fast food is not.
- Neither Jim nor Toby is on fire.
- Either I am hilarious or you have no sense of humor.

Practice Truth Tables

- $p \wedge \sim q$
- $(p \wedge \sim r) \vee (p \wedge r)$
- $(p \wedge \sim q) \vee (\sim q \vee \sim p)$
- $(p \wedge \sim r) \vee (q \vee \sim r)$
- $(p \vee q) \wedge \sim (p \wedge q)$