

## Subset relations

Given any sets $A$ , $B$ , and $C$ :	
1. Inclusion for intersection:	$(A \cap B) \subseteq A$ $(A \cap B) \subseteq B$
2. Inclusion for union:	$A \subseteq (A \cup B)$ $B \subseteq (A \cup B)$
3. Transitive property of subsets:	$(A \subseteq B) \wedge (B \subseteq C) \rightarrow A \subseteq C$

## Set identities

Given any sets $A$ , $B$ , and $C$ , the universal set $U$ and the empty set $\emptyset$ :	
1. Commutative laws:	$A \cap B = B \cap A$ $A \cup B = B \cup A$
2. Associative laws:	$(A \cap B) \cap C = A \cap (B \cap C)$ $(A \cup B) \cup C = A \cup (B \cup C)$
3. Distributive laws:	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4. Intersection with $U$ (identity):	$A \cap U = A$
5. Double complement law:	$(A')' = A$
6. Idempotent laws:	$A \cap A = A$ $A \cup A = A$
7. De Morgan's laws:	$(A \cup B)' = A' \cap B'$ $(A \cap B)' = A' \cup B'$
8. Union with $U$ (universal bounds):	$A \cup U = U$
9. Absorption laws:	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$
10. Alternative representation for set difference:	$A - B = A \cap B'$

## Properties of $\emptyset$ and the universal set

Given any sets $A$ , $B$ , and $C$ , the universal set $U$ and the empty set $\emptyset$ :	
1. Definition of empty set:	$(\forall \text{ sets } A) [A = \emptyset \leftrightarrow (\forall x \in U) [x \notin A]]$
2. The empty set is a subset of every set:	$(\forall \text{ sets } A) [\emptyset \subseteq A]$
3. Union with $\emptyset$ :	$A \cup \emptyset = A$
4. Intersection and union with complement:	$A \cap A' = \emptyset$ $A \cup A' = U$
5. Intersection with $\emptyset$ :	$A \cap \emptyset = \emptyset$
6. Complement of union and $\emptyset$ :	$U' = \emptyset$ $\emptyset' = U$
7. Every set is a subset of the universal set:	$(\forall \text{ sets } A) [A \subseteq U]$