CMSC 250 Homework #3, DUE: Wed. 2/27

Be sure to write legibly or we will not grade your answer. When submitting to GradeScope, failure to correctly identify the proper page numbers for each question will result in a grade of 0.

This homework will require you to write proofs. Don't forget that each proof must begin with a statement that you claim to be true. "Claim: ....". After that, we should see the word "Proof" followed by a correct and carefully written proof, following the guidelines demonstrated in lecture.

1. [40 pts] Give a "constructive" proof for each of the following:

   a. There exist $a, b, c \in \mathbb{N}$ (all distinct) such that $a + b^2 = c^3$.

   b. There exists $n \in \mathbb{N}$ such that n can be written two different ways as the sum of two primes. (Obviously, writing something like $8 = 5 + 3$ and also $8 = 3 + 5$ does not count!)

   c. There exists $n \in \mathbb{N}$ such that $3^n + 2$ is composite.

   d. There exist $a, b, c \in \mathbb{N}$ such that a is a factor of bc, but a is not a factor of b and a is not a factor of c.

2. [18 pts]Prove the following using the method of "exhaustion".

   a. For all integers, n, such that $1 \leq n \leq 10$, $n^2 - n + 11$ is a prime.

   b. There are no primes strictly between 113 and 127. (Show some factors here.)

3. [42 pts]Prove the following using the method that relies on "Universal Generalization". (Typically, these proofs start with something like: Let $a \in \mathbb{N}$, selected arbitrarily...)

   a. $(\forall n \in \mathbb{N}^+)[n^2 + 4n + 3$ is composite$]$.

   b. $(\forall n \in \mathbb{N})[n^2 + n$ is even$]$. Hint: Consider two cases: n is even or n is odd.

   c. For all $n \in \mathbb{N}$, if n is the product of four consecutive integers, then $n + 1$ is a perfect square.