CMSC 250 Homework #4, DUE: Wed. 3/6

For each of the following statements, either prove the statement or give a counterexample that shows the statement is false. **We will use the (non-standard) notation $\mathbb{I}$ to represent the irrational numbers.**

Each problem is worth 10 points.

1. For all $m \in \mathbb{N}^{>2}$, $m^2 - 1$ is composite.
2. For all integers a and b: If ab is even then a is even or b is even.
3. For all integers a, b, and c: If $a|c$ and $b|c$ then $ab|c$.
4. For all integers a, b, and c: If $a|b$ and $a|c$ then $a|(b-c)$.
5. For all integers a and b: If $a|12b$ then $a|12$ or $a|b$.
6. For all integers a, b, and c: If $a|(b+c)$ then $a|b$ or $a|c$.
7. For all integers m, if 7 is a factor of m then 7 is not a factor of $m+6$.
8. $(\forall x \in \mathbb{I}^{+})[\sqrt{x} \in \mathbb{I}]$
9. $(\forall x, y \in \mathbb{Q})(\forall z \in \mathbb{I})[\text{If } y \neq 0 \text{ then } x + yz \in \mathbb{I}]$
10. $log_5(2) \in \mathbb{I}$. Hint: Consider using the Fundamental Theorem of Arithmetic.