Homework 5 – Due **Friday 3/15**

1. What is the remainder when 7^{5555} is divided by 6? Explain briefly.
2. What is the remainder when 5^{7777} is divided by 6? Explain briefly.
3. What is the remainder when 5^{6666} is divided by 6? Explain briefly.
4. Let k be the number of people on the planet Earth. What is the remainder when $15k^{200} + 6(k+2)^{71} + 302$ is divided by 3? Explain briefly.
5. Use modular congruence (mod 2) to decide whether or not the following number is even or odd: $722^{77} - 333^{99}(55^{100})$
6. An "Equivalence Relation" is reflexive, symmetric, and transitive. Prove that modular congruence is an equivalence relation, by proving the following:
   a. Prove that modular congruence is reflexive: $\forall x \in Z, \forall n \geq 1[x \equiv_n x]$
   b. Prove that modular congruence is symmetric: $\forall x, y \in Z, \forall n \geq 1[x \equiv_n y \rightarrow y \equiv_n x]$
   c. Prove that modular congruence is transitive:
      $\forall x, y, z \in Z, \forall n \geq 1[(x \equiv_n y \;\; and \;\; y \equiv_n z) \;\rightarrow\; x \equiv_n z]$
7. Prove that for all natural numbers, n: n is not congruent to $n^2 - 4$ (mod 9)
8. Prove that for all natural numbers, n: n is divisible by 9 if and only if the sum of the digits of n is divisible by 9. Hint: We did a similar example in class.
9. Show that if n is a natural number and n is congruent to 3 (mod 4) then one of the prime factors of n must also be congruent to 3 (mod 4).
10. Prove that for all integers n and d: $d|n \;\leftrightarrow\; n = d \cdot \lfloor n/d \rfloor$
11. Prove that for any real number x: If x is not an integer, then $\lfloor x \rfloor + \lfloor -x \rfloor = -1$